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A Non-Parametric Approach to Change-Point Detection in Cross-Asset Correlations

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A NON-PARAMETRIC APPROACH TO CHANGE-POINT DETECTION

IN CROSS-ASSET CORRELATIONS.

by

L. KAILI DIAMOND

(Under the Direction of Patricia Humphrey, Ph.D.)

ABSTRACT

In this thesis we explore the problem of detecting change-points in cross-asset correlations using a non-parametric approach. We began by comparing and contrasting several common methods for change-point detection as well as methods for measuring correlation. We finally settle on a statistic introduced in early 2012 by Herold Dehling et.al. and test this statistic against real world financial data. We provide the estimated change-point for this data as well as the asymptotic p-value associated with this statistic. Once this process was complete we went on to use simulated data to measure the accuracy, power, and type 1 error associated with this new statistic. Finally, we were able to draw conclusions on the functionality and usefulness of this statistic.

Index Words: cross-asset correlation, non-parametric, change-point, diversification

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INTRODUCTION

Purpose of the Study

Trying to predict changes in financial markets is a problem as old as the markets themselves. Many mathematicians and financial analysts have spent countless hours trying to find mathematical indicators of when a market or specific asset is going to shift in value. With the recent economic downturn which began in late 2007, this problem has once again become one of the most important issues on many investors' minds. While there are countless factors causing shifts in markets, the idea is that the asset value itself should give us some indication of an approaching change. Many people have worked on the problem of price changes within markets as a way to later hedge against losses.

There are risks associated in any financial contract that you enter. Therefore, hedging practices are used to help offset the effects of negative events. For instance, when a person buys a car or a house they generally buy insurance to compensate for possible losses. The risks themselves cannot be eliminated, but the negative financial effects that can be caused by those risks can be reduced. In the case of investment risks the practice of hedging becomes more complicated than simply paying a yearly fee in return for compensation in the event of a loss. Instead, investors strategically use financial instruments to offset financial downturns in other instruments. In other words, they hedge against investment risk by making more investments.

A portfolio is a collection of investments held by a person or company. Asset allocation within a portfolio is the way in which a person spreads the total investment amount among the individual investments within that portfolio. The idea of diversification as a way to hedge against losses uses strategic asset allocation. Specifically, an investor intentionally invests in two or more assets that are negatively correlated or not correlated at all. In plain terms, the investments are made in assets whose prices either trend opposite to each other or have no relationship to each other.

Diversification was first introduced as a way to hedge against portfolio risk by Harry Markowitz in his 1952 publication, *Diversification and Portfolio Risk.* Markowitz went on to win the Nobel Prize in Economics in 1990. His idea blossomed and is now standard practice as a way to reduce a portfolio's risk exposure.

In diversification, both negatively correlated and uncorrelated assets can be used. However, when two assets are strongly negatively correlated, they are both likely to respond to the same event. In this case, the value of one asset (asset A) will move up while the value of the other asset (asset B) moves down. While this is mitigates the loss in B, any time one asset is growing in value, the other is theoretically going to fall in value. In the case of no correlation, if an event happens to make the value of asset A go up or down, that same event should have either no effect or an unpredictable effect on asset B. Asset B should continue to perform as it did before the event. While the lack of correlation might not offset negative movement as strongly as negative correlation would, the investor will not automatically lose money in one asset because the other is doing well and can actually have growth (or decay) in the value of both assets.

Improper diversification methods, including investing in strongly correlated assets or investing in a single asset, can prove to be disastrous for the investor. The old adage "putting all your eggs in one basket" is a perfect analogy for this type of investing. A prime example of this was during the Enron scandal. Many of Enron's employees had the majority, if not all, of their retirement savings tied up in Enron stock. Therefore, when the company went bankrupt and the stock was worthless these employees lost everything that they thought they had saved for retirement. If an employee had invested prudently and had a truly diversified portfolio, only the funds in the Enron stock would have been lost while funds invested in other assets would be safe (unless the other asset had a strong positive correlation with Enron stock).

Diversification of a portfolio is a continuous process. The correlations among assets can change as time goes on as well as the percentage of funds in each asset due to the outperformance of one asset over another. "In particular, correlations among stock returns seem to increase in times of crisis, as evidenced by the most recent joint downturn in stock markets worldwide."[8] For this reason, reallocation of assets is a constant consideration. Prudent investors periodically check to see that their diversification strategies are still effective. In the case of correlations, this information can only be found in retrospect. This is to say that an investor can only look at previous data to determine whether the assets in question are correlated. Because statistical significance of the correlation depends on the number of data points being considered, the sensitivity to a change in correlation decreases as the number of data points increases. For this reason, an investor's portfolio may have been exposed to correlation risk for an extended period of time before he or she is aware that there is a problem. Our goal is to find a way to more quickly, accurately, and efficiently detect a positive change in asset correlations.

Statistics have long been used as a powerful and important tool in the analysis of financial data. Finding the expected return and volatility of an asset are two very important areas of financial analysis. Stochastic models are often used to perform these analyses. However, the use of stochastic models is more for the ease of calculation and in many cases does not accurately portray real world data. This means that, while the calculations are made easy, if the correct model is not applied the results are basically meaningless. Many times there isn't a stochastic distribution that adequately portrays the data. Two of the biggest problems with financial data are that it tends to have heavier tailed distributions than the commonly used normal distribution and each data point is not independent of all of the rest. Looking at the price of an asset, one can easily see that the price on one day has some dependence on the price the previous day. This is why nonparametric approaches to these analyses are gaining in popularity. Non-parametrics allow the financial analyst the freedom of not assuming a known underlying distribution to the data but instead rely on assumptions that are somewhat less restrictive.

While nonparametric statistics can provide an eloquent way to find the needed statistics, the calculations themselves can be tedious and many times are computationally intensive and impractical for “hand” calculation for even moderate size data sets. Thanks to improvements in computing power and availability, the tediousness of these calculations is no longer an issue. This is why nonparametric analysis is growing rapidly in popularity in the financial world.

Another consideration when looking at financial data is that most of the time they are time series. That is, each data point or measurement is taken sequentially over time. Pricing data can be observed daily, weekly, monthly, or at any other time period. This means that these data fail to fulfill one of the most critical assumptions of standard methods - namely, that observations are independent of each other. One of the biggest interests in time series analysis is in forecasting. This is seen in the use of time series in the financial realm.

Classical time series methods such as linear regression, ARMA models, and the Black-Scholes model do not take diversification into consideration. They are strictly concerned with one, and only one, asset. This can lead investors to make decisions based on information about one asset that does not necessarily benefit the portfolio as a whole. These classical approaches also depend on fitting the data to a model, an idea that often falls short in accuracy when dealing with financial data. These are some other reasons that nonparametric methods are rapidly growing in the world of financial forecasting.

Now that we have discussed the financial and mathematical background to our investigation, we can move toward making some inferences and discussing possible solutions to the questions that we wish to answer: Can we locate a specific change point in the correlation of two assets using non-parametrics? If so, how much data is needed to detect this change-point? And finally, how robust are these methods?

When trying to diversify a portfolio, investors start by determining two or more assets that are uncorrelated or negatively correlated and invest in both assets. "Historically, diversification benefits resulted in significant investment interest for commodities." [6] However, there seems to have been a shift in correlations over the past few years leading to much higher correlations between commodities and equities as well as commodities amongst themselves. [6]

We decided to look at the weekly price data of two commodities over the time period of 2003 until the current date (March 2012). The idea was to choose two commodities that would likely be unrelated in a market sense. The first asset of interest that came to mind for us was the price of a barrel of oil. Oil in particular seems to always be of interest to investors as well as world economists. The price of oil can be influenced by many factors including unrest in the middle east, new laws and regulations set by governments or OPEC, Organization of the Petroleum Exporting Countries, and even speculations by investors of a change approaching in just about anything that is related to the world economy. The next thing that came to mind was the price of a ton of soybeans. Factors that would influence the price of soybeans include time of year, weather (such as drought) in soybean producing countries, and the price of storage. One could conceive that oil may have a slight impact on soybean futures because the equipment used to farm soybeans generally uses gasoline or fuel powered engines. However, this seemed like it would have a minimal impact on the overall value of soybeans.

We were able to download weekly price per barrel from the Energy Information Administration web. Next, we were able to gather weekly data on the price per metric ton (1000 kilograms) of soybeans from wikiposit.com. We took the data over the same time frame as the oil prices and then looked at graphs of each of the two commodities prices.

Figure 1.1 Weekly Oil Price 01/03/2003 - 03/16/2012

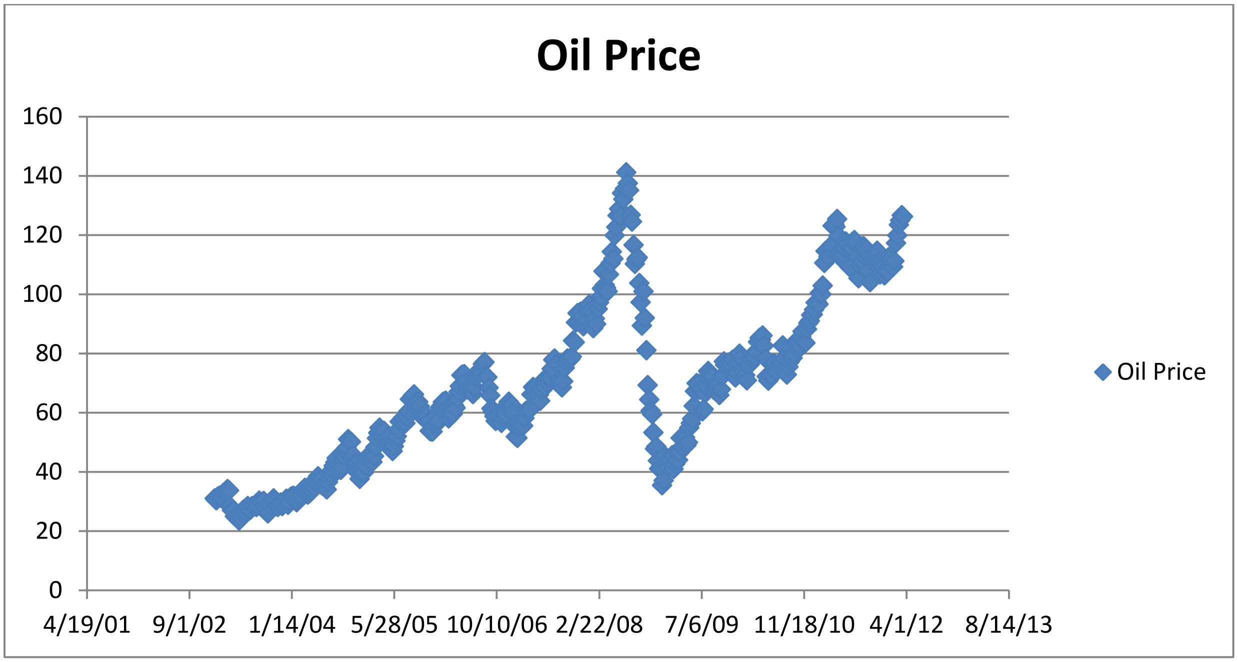
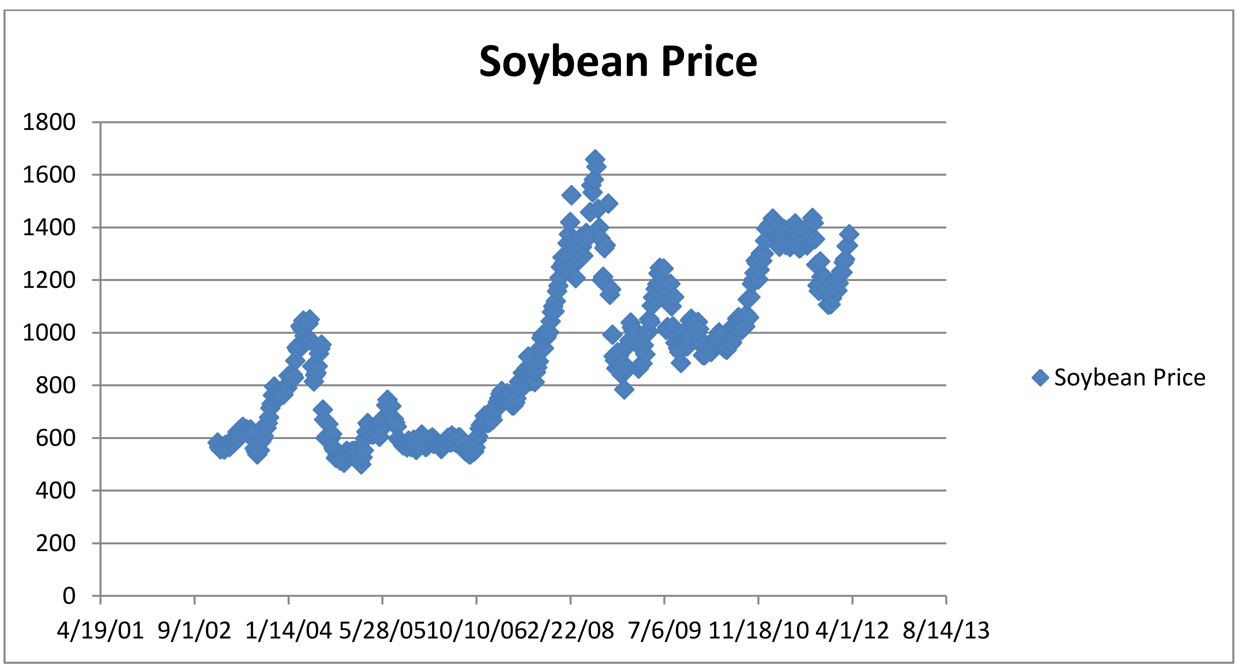


Figure 1.2 Weekly Soybean Price 01/03/2003 - 03/16/2012



By simply looking at the graphs, it appears that the price patterns of these two commodities are completely different from 2003 up to 2007. Then, in late it appears that the prices of both commodities rise sharply before abruptly falling in 2008. This is exactly the type of pattern that we were hoping for since this suggests a change from the two prices being relatively uncorrelated to becoming strongly correlated. For this reason we decided to choose these commodities as our real world data to analyze.

CHAPTER 2

A LOOK AT COMMONLY USED METHODS FOR CORRELATION CALCULATION AND CHANGE-POINT DETECTION

2.1 Cross correlation versus autocorrelation

The word *correlation* is used often. However, there is a distinct difference between *correlation, cross-correlation,* and *autocorrelation.* The *ordinary correlation statistic* is a measure of the linear relationship between two numeric variables. An example of this would be the relationship between a person's height and weight. The *ordinary correlation coefficient* is defined as

$$r = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{(n-1)s_x s_y} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2}\ \sqrt{\sum_{i=1}^{n}(y_i - \bar{y})^2}}.$$

This statistic tells us whether there exists a linear relationship between two variables within a single data set. The ordinary correlation coefficient can range between -1 and 1 with 0 implying that there is no linear relationship between the two variables. Note that a correlation can be 0 (or close to 0) even when two variables are strongly related in a nonlinear fashion, such as a parabolic curve.

An *autocorrelation* is the correlation between a variable and its prior values. In a time series, a non-zero autocorrelation implies that the current value of the series is dependent on past values. The general formula for a *lag k autocorrelation*, with k being the number of time periods that the second series is "lagged", statistic is

$$r_k = \frac{\sum_{i=k+1}^{n}(y_i - \bar{y})(y_{i-k} - \bar{y})}{\sum_{i=1}^{n}(y_i - \bar{y})^2}$$

and has properties similar to the ordinary correlation coefficient.

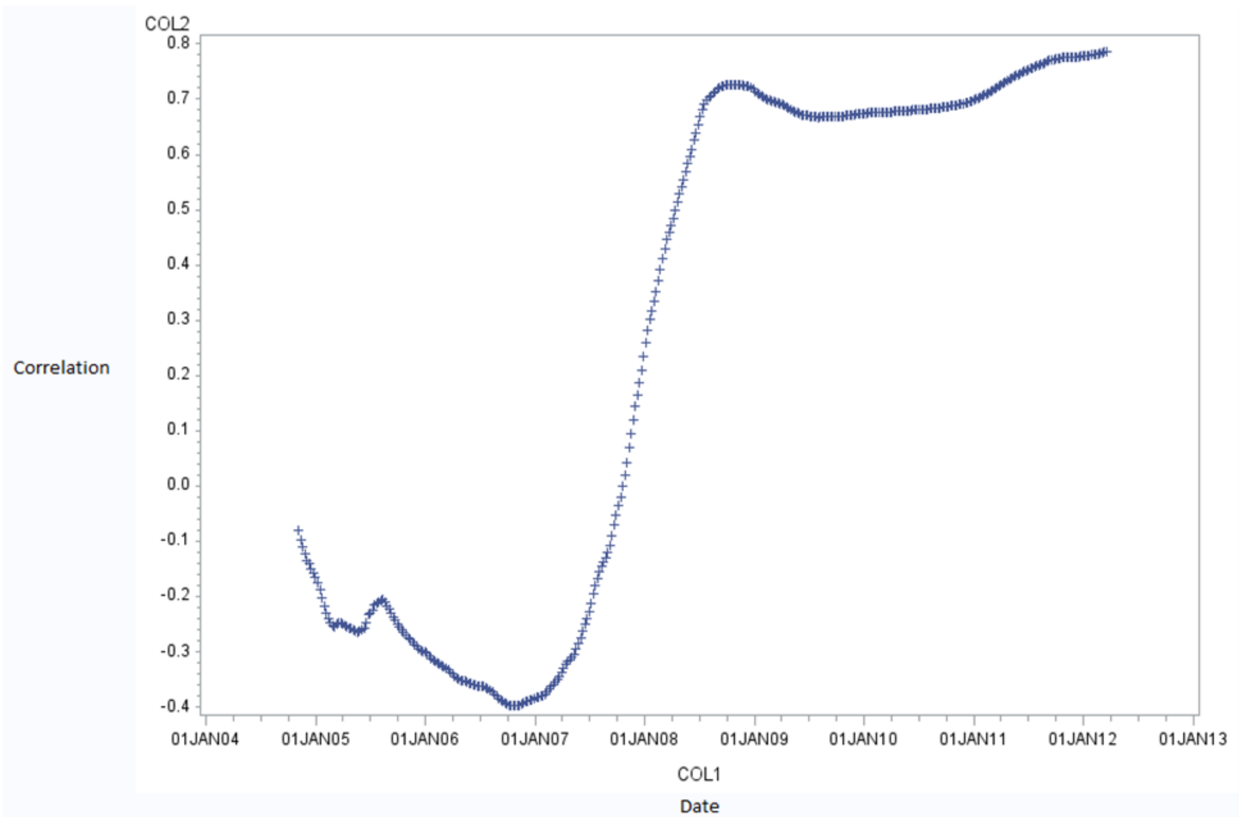
Cross-correlation is similar in calculation to the ordinary correlation coefficient except that x and y no longer represent two variables in a single data set but rather two data points from separate time series. Cross-correlation can also be computed using a lag k approach as follows

$$r_k = \frac{\sum_{i=1+k}^{n}(x_i - \bar{x})(y_{i-k} - \bar{y})}{\sqrt{\sum_{i=1+k}^{n}(x_i - \bar{x})^2}\,\sqrt{\sum_{i=1+k}^{n}(y_{i-k} - \bar{y})^2}}.$$

Using this we can see if two separate time series have a relationship. It is important to recall that correlation does not imply causation. In other words, two correlated time series are not an indication that one depends on the other but simply that they react to the same events. This is the type of correlation that investors are very interested in for diversification purposes.[1]

Even though it appeared by looking at the graphs of the oil and soybean prices that they went from being relatively uncorrelated to being strongly correlated, the only way to know this for sure is to look at the graph of their cross correlation. Therefore, we did just that and found that we had been correct.

Figure 2.1 Cross-correlation of Oil and Soybeans 01/03/2003 – 03/16/2012



We decided to start the graph in mid-2004 even though our data set started at the beginning of 2003. This is because cross correlation is sensitive to the number of data points used. Therefore, we wanted to have enough historical data to accurately portray the true correlation between these assets before determining the strength of the cross correlation.

As can be seen in the graph, the cross correlation among oil prices and soybean prices started out slightly negative in 2004 and continues on this negative trend until 2007. Then, the correlation took a drastic turn in the positive direction taking the two assets from a negative correlation of about -0.4 at the beginning of 2007 to a very strongly positive correlation of about 0.7 in 2009.

2.2 Change-Points

A change-point is the point at which properties of a time series change. [7] Change point detection has many real world applications in medicine, finance, physics, and other fields. Change-points can only be detected in retrospect. In other words, a change-point can't be detected unless it has already occurred through a change in slope, correlation, or autocorrelation. The real question is how much data is needed to detect a change-point. In finance, as in any other field, early detection of a change would be very valuable. There are many different ideas and ways to detect change-points in data.

Quality control techniques such as control charts and cumulative sum charts were initially considered as a way to locate the change point(s) in our financial time series data. Control charts are a traditional way to detect changes in a process due to 'special causes' such as a change in a materials supplier in the case of manufacturing, rather than what would be considered common variation or random "noise". The stability or instability of the process can then been shown graphically. One benefit to control charts is that they can be updated with each new data point. At first, looking at the problem of detecting a change in correlation using control charts seemed like a reasonable option. We want two assets to be uncorrelated to make our diversification methods effective. Therefore, a window around 0 correlation could be considered 'in control' for our time series. Cumulative sum charts (CUSUM) were introduced by E. S. Page from the University of Cambridge. These charts are another method to detect a change in some parameter of interest, Θ. The process involves summing samples from a process X_n and the assigned weights ω_n (usually the likelihood function) as follows:

$$S_n = 0$$

$$S_{n+1} = \max(0, S_n + X_n - \omega_n).$$

The change is detected by a value of S_n exceeding a given threshold for the process. However, these detection methods generally rely on a time series being stationary, independent and identically distributed (iid), and at least the distribution before the change being known. Financial data cannot be assumed to be stationary and in fact is usually not. Think of long term changes in the Dow Jones Industrial Average, for example. Financial time series data is not iid. In fact, as has been previously stated, data in financial time series are generally dependent on previous values. Also stochastic distributions rarely accurately model financial time series data. With these assumptions not being met, the idea of using quality control methods to detect a change in asset price correlations was abandoned.

With the fairly recent increase in computing power, non-parametric methods seem to be the new horizon for financial data analysis. There are several methods of testing for independence of bivariate data sets non-parametrically. They include a distribution-free test for independence based on signs known as Kendall's Tau and a distribution-free test for independence based on ranks known as the Spearman Coefficient.

2.3 Kendall's Tau

Suppose we are studying a sample of *n* pairs (X_1,Y_1), (X_2,Y_2),…(X_n,Y_n) from a continuous bivariate density. Kendall's Tau uses the idea of concordant and discordant pairs. A concordant pair, (X_i,Y_i), is a pair of observations that move in the same direction. [4]

$$X_{i-1} < X_i \textit{ and } Y_{i-1} < Y_i$$

or

$$X_{i-1} > X_i \textit{ and } Y_{i-1} > Y_i$$

A discordant pair is a pair of observations that move in different directions.

$$X_{i-1} < X_i \textit{ and } Y_{i-1} > Y_i$$

or

$$X_{i-1} > X_i \textit{ and } Y_{i-1} < Y_i$$

We start by assuming that (X_i,Y_i) are iid from some joint distribution F_{xy}. By definition, this means that $F_{xy}(x,y)=F_x(x)F_y(y)$. Define the null hypothesis as $H_0= F_{xy}(x,y)=F_x(x)F_y(y)$ and our τ as $\tau=2*P((Y_i-Y_{i-1})(X_i-X_{i-1})>0)-1$. One can easily see that X independent of Y implies that $\tau=0$. By the law of contraposition $\tau\neq0$ implies that X is not independent of Y. A warning: this is an "if" and not an "if and only if" statement. Therefore, $\tau=0$ does NOT imply that X and Y are independent. We expect $\tau > 0$ if there are mostly concordant pairs implying a positive association and $\tau < 0$ if there are mostly discordant pairs implying a negative association. [4] The value of τ is not a measure of association but rather a probability estimate. To compute the Kendall's sample correlation statistic for Kendall's sign test simply subtract the number of discordant pairs from the number of concordant pairs in the data set, i.e. K = C - D. For the exact test, we reject H_0 if:

Right Tailed: $K \geq k_\alpha$

Left Tailed: $K \leq -k_\alpha$

Two-Tailed: $|K| \geq k_{\alpha/2}$

For the right tail test, the alternative $H_1 = \tau>0$ implies a positive correlation. For the left tailed test, $H_1 = \tau<0$ implying a negative correlation. Finally, the two-tailed test has $H_1 = \tau\neq0$. The two-tailed test simply implies an associate without specifying positive or negative. There are three types of Kendall's τ, τ_a, τ_b, and τ_c. τ_a does not adjust for ties. τ_b does make an adjustment for ties. This is the only one that SAS will compute. The type τ_c is generally used for rectangular matrices. [4] Since τ is a probability that cannot be found explicitly, there is an estimation for τ, $\hat{\tau}$ [5].

$$\hat{\tau} = \frac{2K}{n(n-1)} = \frac{2K}{\binom{n}{2}}$$

For the large sample approximation of K:

$$K^* = \frac{K - E_0(K)}{\sqrt{\{var_0(K)\}}} = \frac{K}{\sqrt{\{\frac{n(n-1)(2n+5)}{18}\}}}$$

Here, we reject H_0 if:

Right Tailed: $K^* \geq z_\alpha$

Left Tailed: $K^* \leq -z_\alpha$

Two-Tailed: $|K^*| \geq z_{\alpha/2}$ [5]

In SAS, proc corr will return the large sample approximation for Kendall's Tau correlation statistic along with the associated p-value. There is an asymptotic confidence interval for Kendall's Tau by Samara-Randles, Fligner-Rust, and Noether. [5] However, like many non-parametric calculations, this calculation is tedious and the formula has been omitted for brevity.

2.4 Spearman's Rank Test

The initial assumptions for Spearman's Rank Test are the same as for Kendall's tau including data being from a bivariate distribution with joint density F_{xy} and marginal distributions F_x and F_y. However, unlike Kendall's Tau, the alternative hypotheses are more general concepts of positive and negative association. For the right tailed test, H_1 = X and Y are positively associated. For the left tailed test, H_1 = X and Y are negatively associated. Finally, for the two-tailed test, H_1 = X and Y are not independent variables.[5]

The test procedure begins by ranking the X and Y separately with $R_i = \text{rank}(x_i)$ and $S_i = \text{rank}(y_i)$. We then define $D_i = R_i - S_i$. To calculate the exact Spearman's Rank Coefficient:

$$r_s = \frac{12\sum_{i=1}^{n}\left\{\left[R_i - \frac{n+1}{2}\right]\left[S_i - \frac{n+1}{2}\right]\right\}}{n(n^2-1)} = 1 - \frac{6\sum_{i=1}^{n} D_i^2}{n(n^2-1)}$$

where $D_i = S_i - R_i$, for $i=1,2,\ldots n$.

Reject the null hypotheses, H_0, as follows:

Right Tail: $r_s \geq r_{s,\alpha}$

Left Tailed: $r_s \leq -r_{s,\alpha}$

Two-Tailed: $|r_s| \geq r_{s,\alpha/2}$

The large sample approximation for r_s is:

$$r_s^* = \frac{r_s - E_0(r_s)}{\sqrt{\{var_0(r_s)\}}} = \sqrt{(n-1)}r_s$$

since $E_0(r_s) = 0$ and $var_0(r_s) = \frac{1}{n-1}$. Now, we reject H_0 if:

Right Tail: $r_s^* \geq z_\alpha$

Left Tailed: $r_s^* \leq -z_\alpha$

Two-Tailed: $|r_s^*| \geq z_{\alpha/2}$

Dominik Wied and Walter Kramer [9] proposed a test statistic for finding a change point based on the Pearson Correlation Coefficient, ϱ, where

$$\rho_t = \frac{Cov(X_t, Y_t)}{\sqrt{Var(X_t)}\sqrt{Var(Y_t)}}.$$

They were testing whether the correlation between X_t and Y_t is constant over time. For their test H_0: $\varrho_t = \varrho_0$ $\forall$ $t \in \{1, \ldots, T\}$ versus H_1: $\exists t \in \{1, \ldots, T-1\}$ such that $\rho_t \neq \rho_{t+1}$ for a constant ρ_0. The proposed test statistic is

$$Q_t(X, Y) = \widehat{D}\ max_{1 \leq j \leq T} \left(\frac{j}{\sqrt{T}} |\hat{\rho}_j - \hat{\rho}_T| \right)$$

where,

$$\hat{\rho}_k = \frac{\sum_{t=1}^{k}(X_t - \bar{X}_k)(Y_t - \bar{Y}_k)}{\sqrt{\sum_{t=1}^{k}(X_t - \bar{X}_k)^2}\sqrt{\sum_{t=1}^{k}(Y_t - \bar{Y}_k)^2}}$$

and

$$\bar{X}_k = \frac{1}{k}\sum_{t=1}^{k} X_t \ and\ \bar{Y}_k = \frac{1}{k}\sum_{t=1}^{k} \bar{Y}_t.$$

Wied et al. specified the scalar $\hat{D}$ which is needed for the asymptotic null distribution. The idea behind this test statistic is that "the test rejects the null hypothesis of constant correlation if the empirical correlations fluctuate too much."[8]

This approach was considered. However, we believed that there was a better approach that did not depend on a parametric statistic such as the Pearson Coefficient. We found an approach based on Spearman's Rho proposed by Wied, Dehling, van Kampen, and Vogle (2011). Here, the previous work of Wied and Dehling using Pearson's Coefficient is altered to be closely related to Spearman's Rho. The test statistic is

$$W = \hat{D}\ max_{1 \leq k \leq n} \left| \frac{k}{\sqrt{n}} (\hat{\rho}_k - \hat{\rho}_n) \right|$$

where $\hat{D}$ is given by the authors but omitted here for brevity. One can immediately see the similarities to the test statistic $Q_t(X,Y)$. This makes sense as the two proposed test statistics share several authors.

While Spearman's Rho and Kendall's Tau are similar in statistical properties, Croux and Dehon (2010) compare the robustness and efficiency at the normal model and conclude that Kendall's Tau is the more favorable option. Dehling shared these feelings as he along with Vogel, Wendler, and Wied authored a paper in March 2012 proposing a new test statistic that is closely related to Kendall's Tau. This is the test statistic that we endeavored to research further.

CHAPTER 3

METHODS

Dehling extended the work that he did with Wied and Kramer to take a non-parametric approach to the problem of locating a change point among asset correlations. He started by changing the test statistic to depend on Kendall's Tau rather than the Pearson Coefficient. Instead of directly calculating the correlation of two time series, Kendall's Tau tells the probability of an association and whether that association is positive or negative. Another argument for the use of Kendall's Tau over Person's Coefficient is that τ "possesses a higher efficiency at heavy tailed distributions." [2] However, Dehling et al. used τ_a and we used τ_b. We did this for several reasons. First, the program that we used (SAS) to find the test statistic and p-value for both our financial pricing data and for our simulations will only compute τ_b. Second, we believe that it is unreasonable not to account for ties. Third, of my financial pricing data which has 481 pairs, only 17 ties occurred in the oil prices and 4 in soybean prices. This means that the difference between τ_a and τ_b will be negligible.

Another inconsistency is in the general way that τ is defined and the way that Dehling et al. defined it. The general definition is, as stated in the previous chapter, $\tau=2*P((Y_i-Y_{i-1})(X_i-X_{i-1})>0)-1$. This is the definition that SAS uses when finding $\hat{\tau}$. Dehling et al. used the definition $\tau=P((Y_i-Y_{i-1})(X_i-X_{i-1})>0)$. This definition is what was used to describe the asymptotic distribution and therefore will not only have an effect on the test statistic but also the asymptotic p-value. Fortunately, this is a linear transformation so we simply performed the inverse linear transformation on the τ's found in our program to fix the discrepancy.

The test statistic proposed by Dehling is

$$\hat{T}_n = max_{1 \le k \le n}\left(\frac{k}{\sqrt{n}}|\hat{\tau}_k - \hat{\tau}_n|\right).$$

The null hypothesis for this test statistic is H_0: there is no change in correlation between the marginal distributions of the pairs (X_i,Y_i) and the alternative is H_1:there is a change in correlation between (X_i,Y_i).

In the paper "An Efficient and Robust Test for Change-Points in Correlation" the authors described and developed the test statistic and the p-value for the statistic, however, they failed to run their test statistic on any real world data. We decided that would be the place to start. Once we had successfully written code in SAS to calculate the $\hat{T}_n$ and p-value, we ran it on real financial data as well as simulated data sets.

The SAS code for our test statistic and p-value worked in steps. First, the data was used to find Kendall's Tau for each paired data point. This was accomplished by building a "new" data set by adding each point to the previous points one at a time and outputting each $\hat{\tau}_k$ into a data set. Once this was accomplished we had our $\hat{\tau}_n$ which was the last entry in our table. Next, the function $\hat{T}_k = \frac{k}{\sqrt{n}} |\hat{\tau}_k - \hat{\tau}_n|$ was applied for each k and saved to the data set. The final step was to sort the data in descending order by $\hat{T}_k$. This made the first entry in our data set the maximum and therefore our test statistic $\hat{T}_n$. Once the test statistic was located, we could continue on to find the asymptotic p-value. This was found by first finding $\hat{D}_n^2$ which is a function of the kernel that was given by Dehling et al. Once we had $\hat{D}_n^2$, we could continue by using the joint distribution, F_{xy}, with $\hat{D}_n^2$ to find the asymptotic p-value for our test statistic. For more information on this process, please refer to Dehling et al.'s "An Efficient and Robust Test for Change-Points in Correlations."

One objective was to use the code to run simulations to determine the efficiency of the test statistic. We started by deciding how to generate the simulation data. It has been shown that "if a test is required for non-zero correlation between two time series, both series should first be filtered to convert them to white noise before computing the cross-correlation function." [3] We used this as a ground zero for generating our simulation data which we fitted into a macro in SAS to loop it through our code a specified number of times (n). This way, we could produce a data set in SAS that had all n outputs including the $\hat{T}_n$, p-value, number of observations, the correlation between the series before the known change point, and the number of observations and correlation after the known change point. With this information, it would be easy to examine just how much data was required for our test statistic to detect a known change, how sensitive it was to change in correlation, and how often it correctly located the change point.

Once we had our real world example, our SAS code, and our method for running simulations, we proceeded with Dehling et al.'s test statistic to establish its power and efficiency. The idea was to determine how much data was needed and how large the change in correlation needed to be for this statistic to have significant power.

CHAPTER 4

RESULTS

Since Dehling et al. had neglected to run any real world data (at least that was published in their paper), we decided to start by doing just that. We imported our data into SAS and found our change point and p-value. According to the test, the change point occurred on February 23, 2007 with a p-value of 5.911×10^{-11}. With a p-value this significant, the null is rejected in favor of the alternative which is that the change point occurs on February 23, 2007. Looking at the graph of our correlation, this change point seemed reasonable. However, without running simulations to help determine accuracy and power, we cannot say for certain that this method of change point detection is valid or know how much data is necessary to find the change point.

Next, we ran simulations where we would know the true change point and control how much change in correlation occurred. Our parameters for the simulations were as follows: n1=number of data points before the change in correlation; n2=number of data points after the change in correlation; r1=the correlation of our x and y before the change; r2=the correlation between x and y after the change; and nsim= the number of simulations to run with the previously mentioned parameters which was always set to 1000. The values of x and y were determined by a random number generator from a standard normal distribution with the parameter that the overall correlation between them remain the close to the given r1 and r2. This information was then put through our original SAS code to determine Kendall's Tau for each (X_k,Y_k) created by the random number generator. The process continued as described in the previous chapter for each of the 1000 simulations by a macro that was written to loop through the code, generating a new data set at the beginning of each loop. Each loop output the Replicate number (the test statistic) and the p-value to a new data set to be stored and used for further examination.

After running each simulation and retrieving the data set we ran proc means to find the average change point (replicate) and the average p-value for that of simulations. From this data set we were also able to determine how many times the test statistic found the change point within ±5 replicates of the true change, what the estimated type 1 error was, and the estimated power of the test statistic for that particular set of parameters at an α=0.10 level. This information yielded the following results:

Table 4.1 Simulation Results for Smaller Samples

n1	n2	r1	r2	Avg. Replicate	Avg P-value	Within 5	Est. Power	Est. Type 1 Error
40	10	0	0.2	21.491	0.1758854	168	0.032	0.322
40	20	0	0.2	26.883	0.182776	244	0.099	0.241
40	30	0	0.2	31.84	0.1765535	249	0.119	0.228
40	10	0	0.4	23.506	0.1675343	247	0.064	0.294
40	20	0	0.4	29.208	0.1585997	385	0.177	0.251
40	30	0	0.4	34.018	0.1412458	385	0.219	0.262
40	10	0	0.5	24.992	0.1627557	300	0.083	0.295
40	20	0	0.5	31.535	0.1312008	445	0.255	0.256
40	30	0	0.5	36.21	0.1071198	506	0.351	0.27
40	10	0.2	0.4	21.838	0.1757083	172	0.038	0.32
40	20	0.2	0.4	26.965	0.177525	265	0.114	0.248
40	30	0.2	0.4	32.15	0.1704857	261	0.114	0.267
40	10	0.2	0.5	22.095	0.1777651	197	0.04	0.301
40	20	0.2	0.5	27.297	0.1736528	297	0.123	0.257
40	30	0.2	0.5	33.175	0.1541243	330	0.18	0.251
40	20	0	0.8	35.78	0.0814251	679	0.495	0.238
52	26	0	0.2	35.979	0.1923689	208	0.068	0.232

For larger data sets we looked at all of the same information except we widened the window of interest from ± 5 around the true change to ± 10.

Table 4.2 Simulation Results for Larger Samples

n1	n2	r1	r2	Avg. Replicate	Avg. P-Value	Within 10	Est. Power	Est. Type I Error
100	50	0	0.4	36.335	0.107399	506	0.375	0.3
150	100	0	0.4	140.219	0.041807	526	0.476	0.413
100	100	0	0.4	98.884	0.054375	541	0.491	0.355
100	50	0	0.8	95.514	0.015133	824	0.818	0.176

One can see that this test lacks both power and accuracy with these small data sets and relatively low changes in correlation. In fact, the test only has significant power once the data set reaches 150 data points and a change in correlation of 0.8.

Other consistencies among the simulations include the test's tendency toward negative bias, i.e. underestimating the true change-point, and the relatively large p-values. The average p-values range from 0.015133 up to 0.192369.

To gain a clearer visual idea of how this test performed, we looked at histograms for both average replicate number and p-value of our simulations. Here we have included some examples of this output.

Figure 4.1 Distribution of the Replicate for Condition 1 (n1=40, n2=10, r1=0.2, r2=0.5)

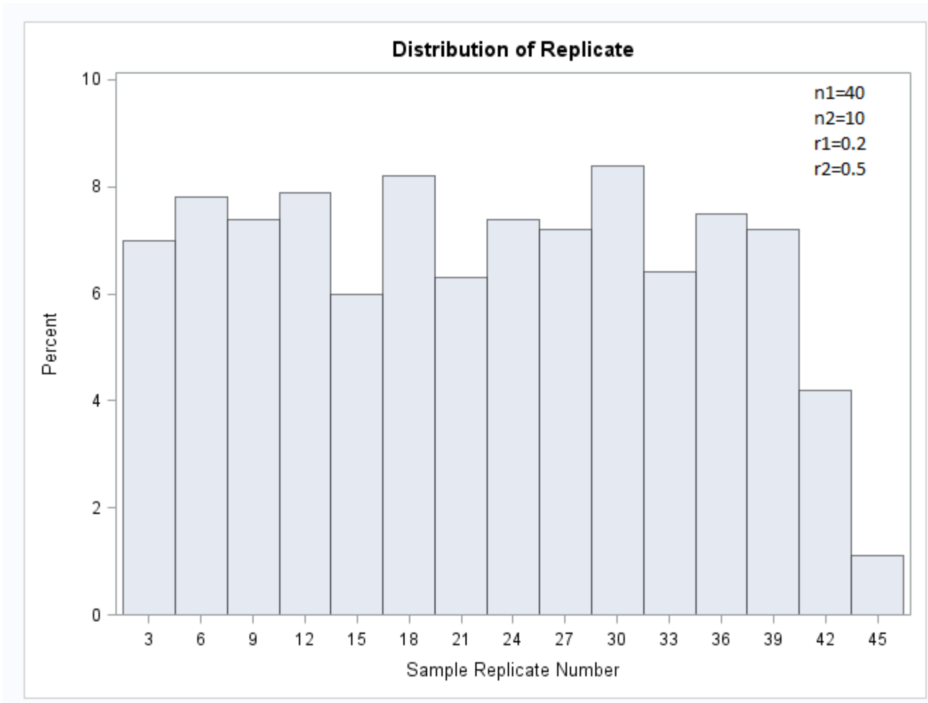


Figure 4.2 Distribution of the P-value for Condition 1 (n1=40, n2=10, r1=0.2, r2=0.5)

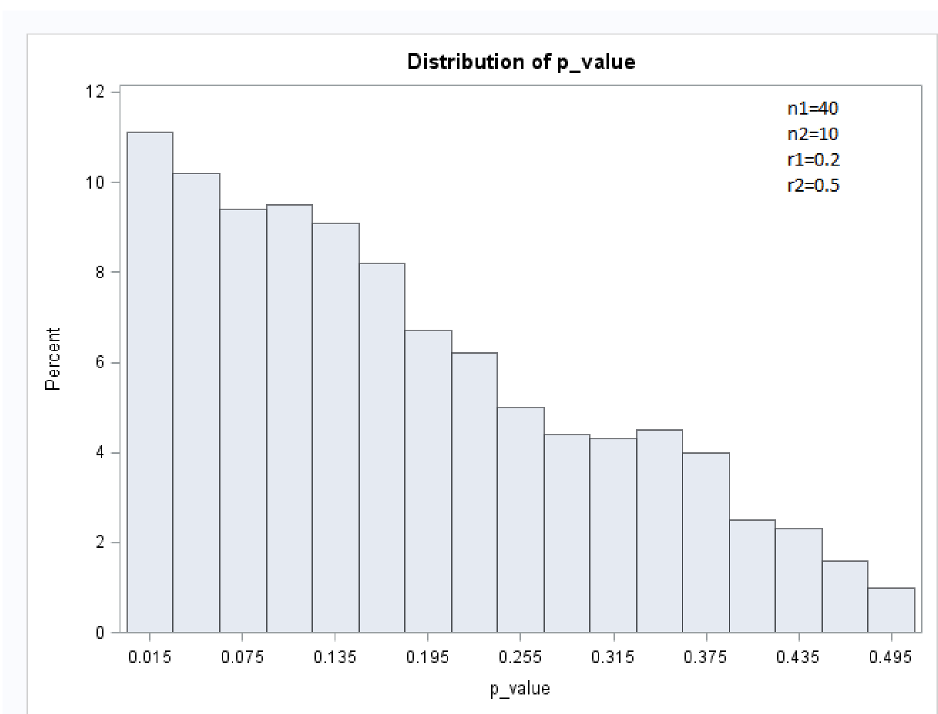


Figure 4.3 Distribution of the Replicate for Condition 2 (n1=40, n2=30, r1=0.0, r2=0.5)

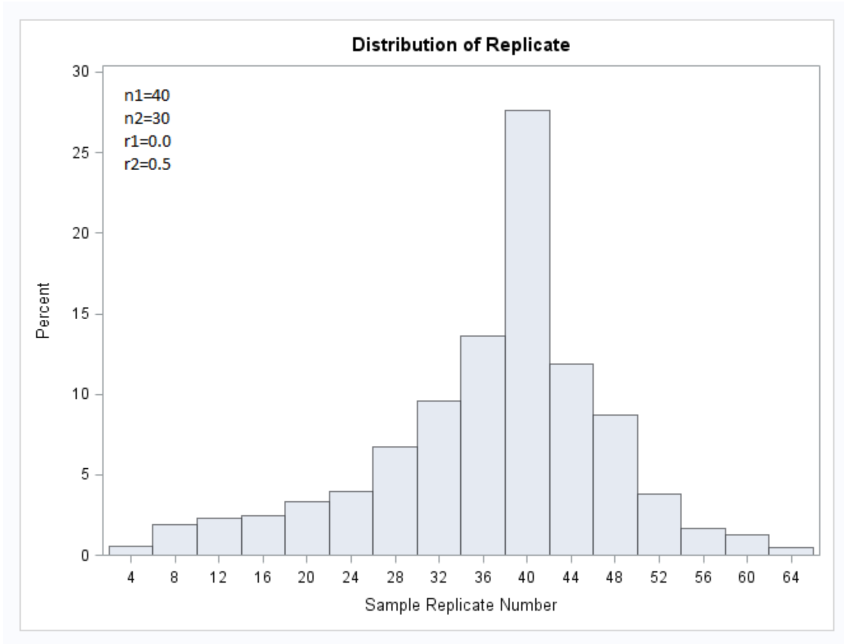
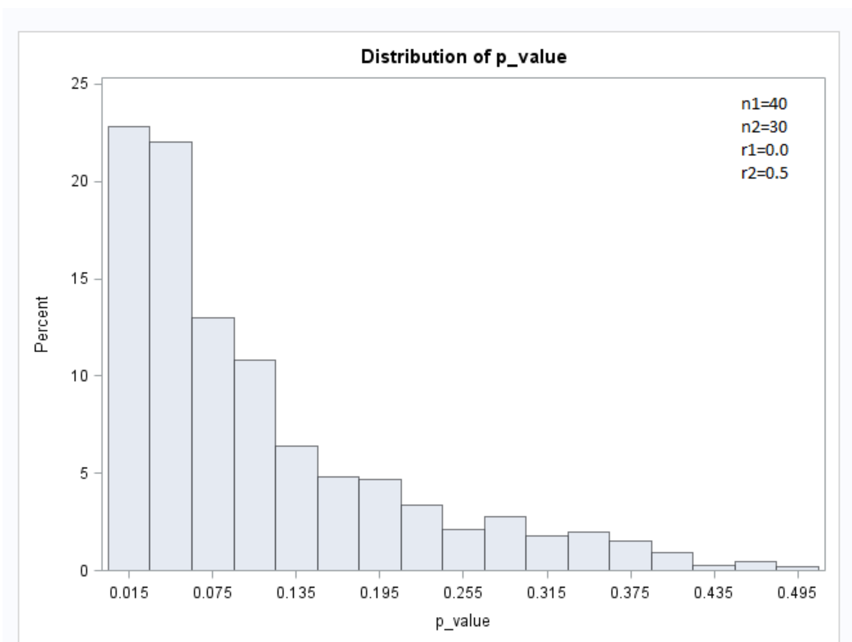


Figure 4.4 Distribution of the P-value for Condition 2 (n1=40, n2=30, r1=0.0, r2=0.5)



From these histograms we can see that there is a very large difference in the test's ability to accurately find a change point and give a significant p-value depending on the amount of change in correlation and the amount of data after the change. This test is basically useless for the first example with the replicate that was chosen as the change point is nearly evenly spread across the entire possible range and approximately 31% of the p-values are insignificant at the 0.10 level. This is not surprising for this data set since non-parametric approaches generally have less power than their parametric counterparts and the data set for this example is extremely small and so is the amount of the change in correlation. There is a very large contrast between the first example and the second. The second example has more data after the change point and a much larger change in correlation. For this example, the test seems to find the actual change point reasonably well and the distribution of the average p-values are heavily skewed right, which is what we want.

It's one thing to look at separate histograms for the replicate and p-values. It's another thing to be able to see how the replicates and p-values coincide. Seeing this interaction would give a visual representation to the idea of power and show us just how much type 1 error we are encountering. The best way to do this was to plot the p-value against the replicate number and then estimate how many times the p-values were below 0.10 as well as how often the test found the change-point within 5 replicates of the actual change. The results were as follows:

Figure 4.5 Plot of P-value by Replicate for Condition 1 (n1=40, n2=10, r1=0.2, r2=0.5)

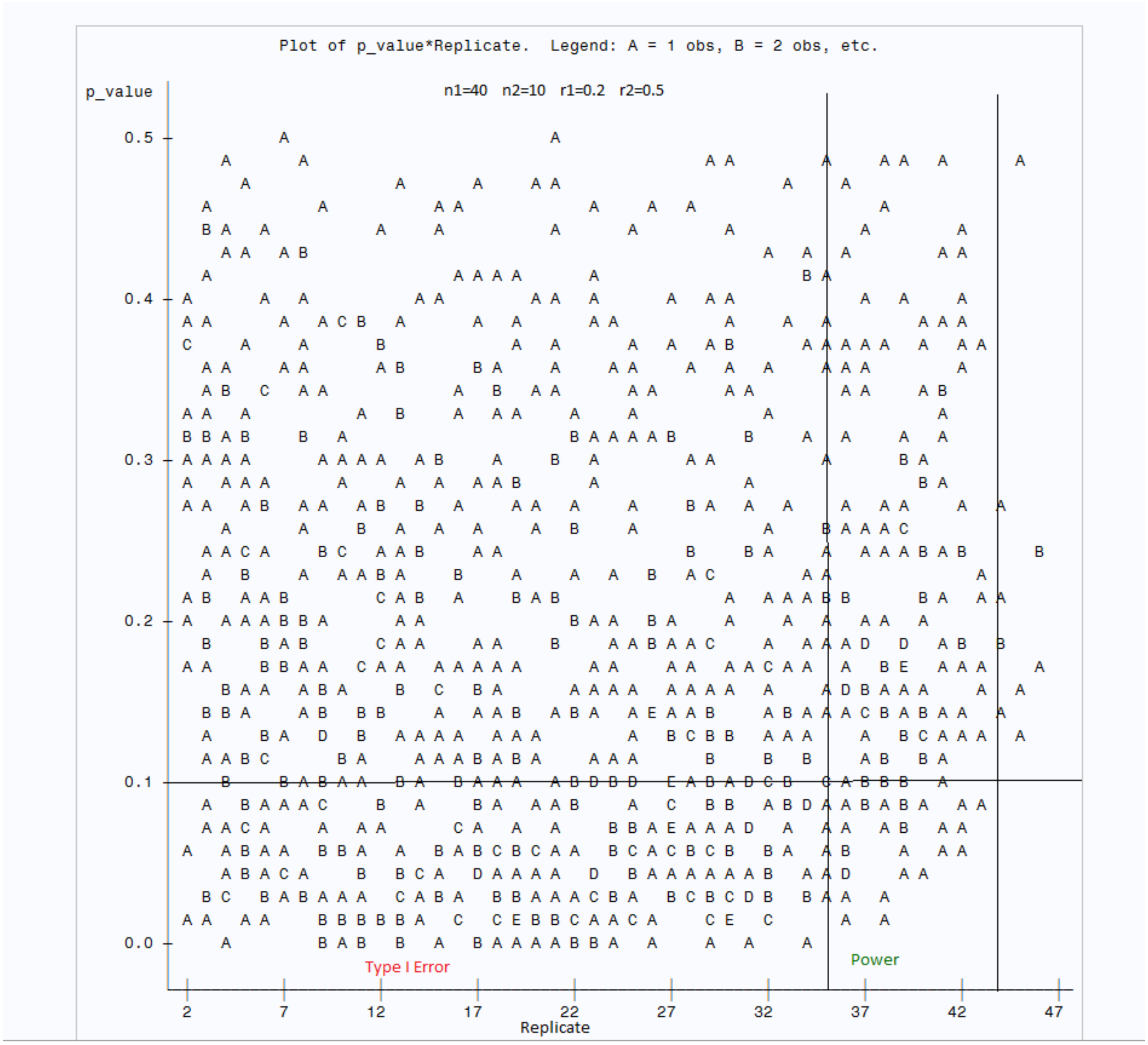


Figure 4.6 Plot of P-value by Replicate for Condition 2 (n1=40, n2=30, r1=0.0, r2=0.5)

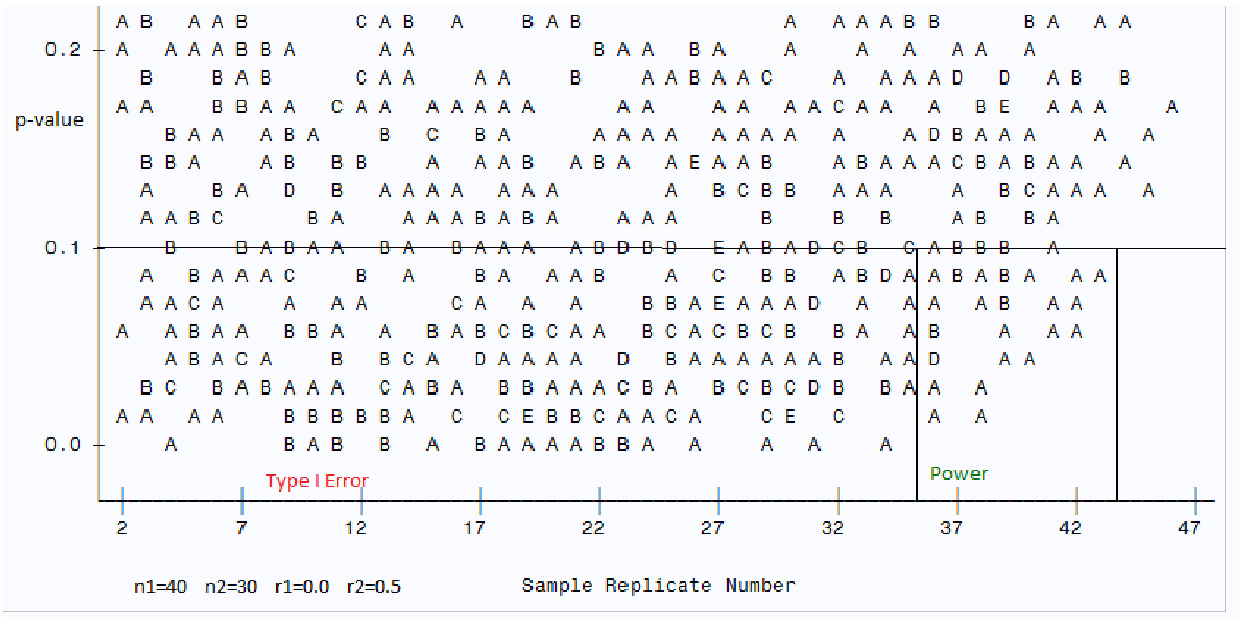
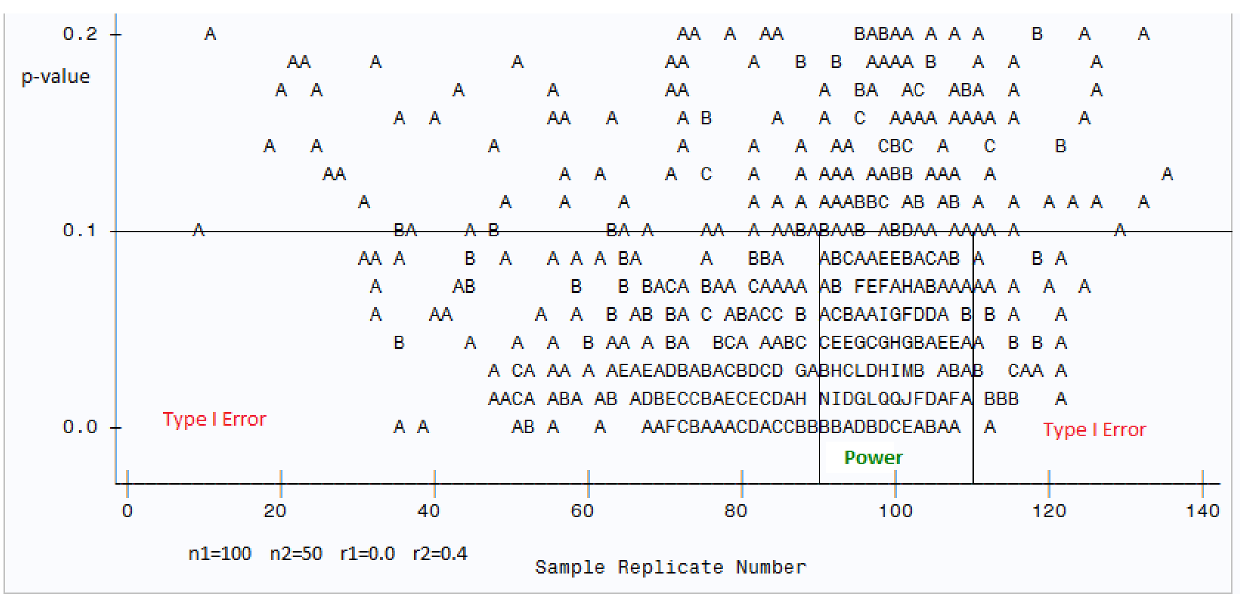


Figure 4.7 Plot of P-value by Replicate for Condition 3 (n1=100, n2=50, r1=0.0, r2=0.4)



It is clear from these illustrations that the power goes up and the type 1 error goes down as the amount of data and the amount of change in correlation go up.

One note to make about this test is that it will always give you an estimate for the change-point, whether or not an actual change has occurred. This has to do with the design of the test. The test statistic is found using a maximum function. This means that there is always at least one solution. It also means that it doesn't have a set "threshold" that must be met before signaling a change. This is important to remember because one cannot simply assume a change has occurred because the test output a replicate number.

CHAPTER 5

CONCLUSIONS AND FURTHER WORK

While we recognize that change-points can only be detected in retrospect, we would like to require minimal data to find the change. This statistic lacks both power and accuracy for data sets that would be of reasonable size for financial data. The test only starts to have significant power once the data set has 100 points before the change and 50 points after as well as a change in cross-correlation of 0.8. This is a problem when you are talking about financial pricing data because pricing data is taken over time intervals such as days or weeks. Not accurately detecting a change until 50 days after the change has occurred is not terrible. However, many long term investors are likely to use weekly or monthly data rather than daily. This is a problem because 50 data points equates to nearly a year when weekly data is observed. This essentially means that an investor's portfolio has been exposed to elevated risk for a year before there is a strong indication that a risk even exists. Also, a cross-correlation of 0.8 would be extremely unacceptable for assets that are supposed to be diversified. In fact, most investors would probably like to reconsider their allocation methods once previously uncorrelated assets become correlated as strongly as 0.3-0.5.

One area that was not explored due to time constraints was the comparison between the power and efficiency of this test statistic as opposed to test statistics that implement Spearman's Rho or Pearson's Coefficient. To truly form an unbiased comparison, these three test statistics would need to run on identical data sets where the true change-point is known. To fully gauge the robustness of these three statistics simulations should be run using several distribution types, instead of only generating standard normal data. This further examination of these test statistics would give us a better understanding of when each one generates significant results and may lead to future improvements in the overall design of our test statistic.

For other fields of study that might be interested in change-point analysis, such as medicine, 150 data points might be considered a fairly small data set. In these sort of studies it is often the case that the change point simply needs to be found and the amount of data required does not have to be as small as in the financial world. For this reason, we feel that this test may have many uses in change-point analysis in general but lacks the necessary power and efficiency to be of use in the financial world, for which it was intended.

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APPENDIX A

SAS Code for Oil Price vs. Soybean Price

```
data work.kaili;
      informat date1 date7. date2 date7.;
      input id date1    oilprice    oilpctchng  Date2 SBPrice     SBpctchng;
      cards;
1     3-Jan-03    30.97       .           3-Jan-03    582.25      .
2     10-Jan-03   30.37       -0.01937    10-Jan-03   564.75      -0.0301
3     17-Jan-03   31.48       0.03655     17-Jan-03   556         -0.0155
4     24-Jan-03   31.81       0.01048     24-Jan-03   569         0.0234
5     31-Jan-03   31.2        -0.01918    31-Jan-03   564         -0.0088
6     7-Feb-03    31.58       0.01218     7-Feb-03    555.75      -0.0146
7     14-Feb-03   32.79       0.03832     14-Feb-03   573         0.0310
8     21-Feb-03   33          0.00640     21-Feb-03   574         0.0017
9     28-Feb-03   33.72       0.02182     28-Feb-03   577         0.0052
10    7-Mar-03    34.11       0.01157     7-Mar-03    564.5       -0.0217
11    14-Mar-03   33.72       -0.01143    14-Mar-03   577.75      0.0235
12    21-Mar-03   28.18       -0.16429    21-Mar-03   576.75      -0.0017
13    28-Mar-03   26.95       -0.04365    28-Mar-03   580         0.0056
14    4-Apr-03    26.93       -0.00074    4-Apr-03    590.75      0.0185
15    11-Apr-03   24.96       -0.07315    11-Apr-03   600.25      0.0161
16    18-Apr-03   24.92       -0.00160    18-Apr-03   622         0.0362
17    25-Apr-03   24.66       -0.01043    25-Apr-03   599         -0.0370
18    2-May-03    23.55       -0.04501    2-May-03    626.25      0.0455
19    9-May-03    24.31       0.03227     9-May-03    636.25      0.0160
20    16-May-03   26.21       0.07816     16-May-03   642         0.0090
21    23-May-03   27.19       0.03739     23-May-03   628.25      -0.0214
22    30-May-03   26.56       -0.02317    30-May-03   624.5       -0.0060
23    6-Jun-03    27.92       0.05120     6-Jun-03    631.5       0.0112
24    13-Jun-03   28.38       0.01648     13-Jun-03   614.5       -0.0269
25    20-Jun-03   26.88       -0.05285    20-Jun-03   629.25      0.0240
26    27-Jun-03   27.19       0.01153     27-Jun-03   631.5       0.0036
27    4-Jul-03    28.4        0.04450     4-Jul-03    626.75      -0.0075
28    11-Jul-03   28.42       0.00070     11-Jul-03   620.75      -0.0096
29    18-Jul-03   28.74       0.01126     18-Jul-03   560.5       -0.0971
30    25-Jul-03   28.02       -0.02505    25-Jul-03   549.5       -0.0196
31    1-Aug-03    28.52       0.01784     1-Aug-03    537.5       -0.0218
32    8-Aug-03    30.19       0.05856     8-Aug-03    548.5       0.0205
33    15-Aug-03   29.6        -0.01954    15-Aug-03   552.25      0.0068
34    22-Aug-03   29.7        0.00338     22-Aug-03   586.25      0.0616
35    29-Aug-03   30.12       0.01414     29-Aug-03   595         0.0149
36    5-Sep-03    28.16       -0.06507    5-Sep-03    608         0.0218
37    12-Sep-03   27.57       -0.02095    12-Sep-03   648         0.0658
38    19-Sep-03   25.9        -0.06057    19-Sep-03   638         -0.0154
39    26-Sep-03   26.49       0.02278     26-Sep-03   656.5       0.0290
40    3-Oct-03    28.38       0.07135     3-Oct-03    678         0.0327
41    10-Oct-03   29.88       0.05285     10-Oct-03   713.5       0.0524
42    17-Oct-03   31          0.03748     17-Oct-03   729.25      0.0221
43    24-Oct-03   29.66       -0.04323    24-Oct-03   762.5       0.0456
44    31-Oct-03   28.46       -0.04046    31-Oct-03   794.25      0.0416
45    7-Nov-03    28.05       -0.01441    7-Nov-03    743         -0.0645
```

46	14-Nov-03	29.01	0.03422	14-Nov-03	773	0.0404
47	21-Nov-03	29.61	0.02068	21-Nov-03	753.5	-0.0252
48	28-Nov-03	28.34	-0.04289	28-Nov-03	756.25	0.0036
49	5-Dec-03	28.93	0.02082	5-Dec-03	760.5	0.0056
50	12-Dec-03	30.17	0.04286	12-Dec-03	776.25	0.0207
51	19-Dec-03	30.87	0.02320	19-Dec-03	764.75	-0.0148
52	26-Dec-03	28.84	-0.06576	26-Dec-03	792	0.0356
53	2-Jan-04	29.78	0.03259	2-Jan-04	792.5	0.0006
54	9-Jan-04	31.5	0.05776	9-Jan-04	790	-0.0032
55	16-Jan-04	31.7	0.00635	16-Jan-04	835.75	0.0579
56	23-Jan-04	31.88	0.00568	23-Jan-04	839.25	0.0042
57	30-Jan-04	30.39	-0.04674	30-Jan-04	819.5	-0.0235
58	6-Feb-04	29.66	-0.02402	6-Feb-04	840	0.0250
59	13-Feb-04	30.12	0.01551	13-Feb-04	829	-0.0131
60	20-Feb-04	31.39	0.04216	20-Feb-04	893	0.0772
61	27-Feb-04	32.27	0.02803	27-Feb-04	942.5	0.0554
62	5-Mar-04	33.73	0.04524	5-Mar-04	933.75	-0.0093
63	12-Mar-04	33.36	-0.01097	12-Mar-04	954	0.0217
64	19-Mar-04	34.56	0.03597	19-Mar-04	1024	0.0734
65	26-Mar-04	33.59	-0.02807	26-Mar-04	1013	-0.0107
66	2-Apr-04	32.23	-0.04049	2-Apr-04	1045.5	0.0321
67	9-Apr-04	32.75	0.01613	9-Apr-04	988	-0.0550
68	16-Apr-04	33.95	0.03664	16-Apr-04	965	-0.0233
69	23-Apr-04	33.7	-0.00736	23-Apr-04	967	0.0021
70	30-Apr-04	34.64	0.02789	30-Apr-04	1034	0.0693
71	7-May-04	36.38	0.05023	7-May-04	1050.5	0.0160
72	14-May-04	37.62	0.03408	14-May-04	969	-0.0776
73	21-May-04	38.43	0.02153	21-May-04	872.5	-0.0996
74	28-May-04	37.95	-0.01249	28-May-04	814	-0.0670
75	4-Jun-04	37.25	-0.01845	4-Jun-04	838.5	0.0301
76	11-Jun-04	35.34	-0.05128	11-Jun-04	847	0.0101
77	18-Jun-04	35.09	-0.00707	18-Jun-04	872	0.0295
78	25-Jun-04	34.71	-0.01083	25-Jun-04	920.5	0.0556
79	2-Jul-04	34	-0.02046	2-Jul-04	939.5	0.0206
80	9-Jul-04	36.51	0.07382	9-Jul-04	954	-0.2584
82	23-Jul-04	39.05	0.03416	23-Jul-04	669.5	-0.0537
83	30-Jul-04	40.66	0.04123	30-Jul-04	599.5	-0.1046
84	6-Aug-04	42.01	0.03320	6-Aug-04	606.75	0.0121
85	13-Aug-04	43.16	0.02737	13-Aug-04	652	0.0746
86	20-Aug-04	44.56	0.03244	20-Aug-04	594.75	-0.0878
87	27-Aug-04	42.23	-0.05229	27-Aug-04	600.5	0.0097
88	3-Sep-04	41	-0.02913	3-Sep-04	614.75	0.0237
89	10-Sep-04	40.6	-0.00976	10-Sep-04	563	-0.0842
90	17-Sep-04	41.55	0.02340	17-Sep-04	553	-0.0178
91	24-Sep-04	45.3	0.09025	24-Sep-04	523.5	-0.0533
92	1-Oct-04	47.11	0.03996	1-Oct-04	534.5	0.0210
93	8-Oct-04	48.09	0.02080	8-Oct-04	528.5	-0.0112
94	15-Oct-04	50.96	0.05968	15-Oct-04	514	-0.0274
95	22-Oct-04	50.5	-0.00903	22-Oct-04	530	0.0311
96	29-Oct-04	50.15	-0.00693	29-Oct-04	527.5	-0.0047
97	5-Nov-04	45.78	-0.08714	5-Nov-04	505	-0.0427
98	12-Nov-04	42.83	-0.06444	12-Nov-04	523.5	0.0366
99	19-Nov-04	40.63	-0.05137	19-Nov-04	550.5	0.0516
100	26-Nov-04	42.78	0.05292	26-Nov-04	548.5	-0.0036
101	3-Dec-04	41.31	-0.03436	3-Dec-04	525.75	-0.0415
102	10-Dec-04	37.6	-0.08981	10-Dec-04	533	0.0138
103	17-Dec-04	39.98	0.06330	17-Dec-04	548.75	0.0295

104	24-Dec-04	41.15	0.02926	24-Dec-04	550.5	0.0032
105	31-Dec-04	39.79	-0.03305	31-Dec-04	547.75	-0.0050
106	7-Jan-05	42.07	0.05730	7-Jan-05	551.75	0.0073
107	14-Jan-05	44.59	0.05990	14-Jan-05	542	-0.0177
108	21-Jan-05	45.08	0.01099	21-Jan-05	516.75	-0.0466
109	28-Jan-05	45.82	0.01642	28-Jan-05	514	-0.0053
110	4-Feb-05	44.04	-0.03885	4-Feb-05	499.5	-0.0282
111	11-Feb-05	43.29	-0.01703	11-Feb-05	525.75	0.0526
112	18-Feb-05	45.22	0.04458	18-Feb-05	552.5	0.0509
113	25-Feb-05	48.23	0.06656	25-Feb-05	597.5	0.0814
114	4-Mar-05	51.33	0.06428	4-Mar-05	622.75	0.0423
115	11-Mar-05	53.09	0.03429	11-Mar-05	656.75	0.0546
116	18-Mar-05	54.85	0.03315	18-Mar-05	649	-0.0118
117	25-Mar-05	53.76	-0.01987	25-Mar-05	628.75	-0.0312
118	1-Apr-05	52.23	-0.02846	1-Apr-05	614	-0.0235
119	8-Apr-05	53.79	0.02987	8-Apr-05	612	-0.0033
120	15-Apr-05	50.43	-0.06247	15-Apr-05	616.25	0.0069
121	22-Apr-05	51.23	0.01586	22-Apr-05	633	0.0272
122	29-Apr-05	51.64	0.00800	29-Apr-05	619.25	-0.0217
123	6-May-05	50.05	-0.03079	6-May-05	635	0.0254
124	13-May-05	48.7	-0.02697	13-May-05	603.75	-0.0492
125	20-May-05	46.98	-0.03532	20-May-05	632	0.0468
126	27-May-05	48.59	0.03427	27-May-05	667.75	0.0566
127	3-Jun-05	50.46	0.03849	3-Jun-05	675.25	0.0112
128	10-Jun-05	51.9	0.02854	10-Jun-05	666.25	-0.0133
129	17-Jun-05	54.17	0.04374	17-Jun-05	724	0.0867
130	24-Jun-05	56.95	0.05132	24-Jun-05	744.5	0.0283
131	1-Jul-05	56.69	-0.00457	1-Jul-05	673.5	-0.0954
132	8-Jul-05	57.98	0.02276	8-Jul-05	675.5	0.0030
133	15-Jul-05	57.43	-0.00949	15-Jul-05	721	0.0674
134	22-Jul-05	56.39	-0.01811	22-Jul-05	669.25	-0.0718
135	29-Jul-05	58.5	0.03742	29-Jul-05	671.75	0.0037
136	5-Aug-05	60.42	0.03282	5-Aug-05	658.5	-0.0197
137	12-Aug-05	64.46	0.06687	12-Aug-05	643	-0.0235
138	19-Aug-05	64.23	-0.00357	19-Aug-05	597.25	-0.0712
139	26-Aug-05	65.68	0.02258	26-Aug-05	591	-0.0105
140	2-Sep-05	66.09	0.00624	2-Sep-05	588.5	-0.0042
141	9-Sep-05	63.24	-0.04312	9-Sep-05	580.5	-0.0136
142	16-Sep-05	61.22	-0.03194	16-Sep-05	571.25	-0.0159
143	23-Sep-05	63.63	0.03937	23-Sep-05	574	0.0048
144	30-Sep-05	62.16	-0.02310	30-Sep-05	573.25	-0.0013
145	7-Oct-05	58.93	-0.05196	7-Oct-05	564.25	-0.0157
146	14-Oct-05	58.59	-0.00577	14-Oct-05	589.5	0.0447
147	21-Oct-05	58.15	-0.00751	21-Oct-05	572.25	-0.0293
148	28-Oct-05	58.5	0.00602	28-Oct-05	565	-0.0127
149	4-Nov-05	58.74	0.00410	4-Nov-05	580.5	0.0274
150	11-Nov-05	56.79	-0.03320	11-Nov-05	593	0.0215
151	18-Nov-05	53.77	-0.05318	18-Nov-05	569.75	-0.0392
152	25-Nov-05	53.57	-0.00372	25-Nov-05	554.25	-0.0272
153	2-Dec-05	53.59	0.00037	2-Dec-05	562.75	0.0153
154	9-Dec-05	56.07	0.04628	9-Dec-05	568.75	0.0107
155	16-Dec-05	58.75	0.04780	16-Dec-05	592.25	0.0413
156	23-Dec-05	56.36	-0.04068	23-Dec-05	613	0.0350
157	30-Dec-05	57.38	0.01810	30-Dec-05	602	-0.0179
158	6-Jan-06	61.72	0.07564	6-Jan-06	600.5	-0.0025
159	13-Jan-06	62.18	0.00745	13-Jan-06	565	-0.0591
160	20-Jan-06	63.54	0.02187	20-Jan-06	568	0.0053

161	27-Jan-06	63.77	0.00362	27-Jan-06	589.25	0.0374
162	3-Feb-06	64	0.00361	3-Feb-06	594.75	0.0093
163	10-Feb-06	61.23	-0.04328	10-Feb-06	582.25	-0.0210
164	17-Feb-06	58.04	-0.05210	17-Feb-06	601.25	0.0326
165	24-Feb-06	59.39	0.02326	24-Feb-06	577.25	-0.0399
166	3-Mar-06	61.06	0.02812	3-Mar-06	593.25	0.0277
167	10-Mar-06	59.5	-0.02555	10-Mar-06	578.25	-0.0253
168	17-Mar-06	62.42	0.04908	17-Mar-06	576.5	-0.0030
169	24-Mar-06	61.61	-0.01298	24-Mar-06	573.75	-0.0048
170	31-Mar-06	64.76	0.05113	31-Mar-06	571.5	-0.0039
171	7-Apr-06	66.93	0.03351	7-Apr-06	557.75	-0.0241
172	14-Apr-06	68.91	0.02958	14-Apr-06	563	0.0094
173	21-Apr-06	72.54	0.05268	21-Apr-06	569.75	0.0120
174	28-Apr-06	72.84	0.00414	28-Apr-06	587.25	0.0307
175	5-May-06	72.92	0.00110	5-May-06	594.5	0.0123
176	12-May-06	70.44	-0.03401	12-May-06	600	0.0093
177	19-May-06	67.84	-0.03691	19-May-06	587	-0.0217
178	26-May-06	68.47	0.00929	26-May-06	582.5	-0.0077
179	2-Jun-06	68.75	0.00409	2-Jun-06	609	0.0455
180	9-Jun-06	67.89	-0.01251	9-Jun-06	585.75	-0.0382
181	16-Jun-06	66.17	-0.02534	16-Jun-06	600.25	0.0248
182	23-Jun-06	68.33	0.03264	23-Jun-06	580.5	-0.0329
183	30-Jun-06	71.82	0.05108	30-Jun-06	594.75	0.0245
184	7-Jul-06	73.45	0.02270	7-Jul-06	602	0.0122
185	14-Jul-06	73.99	0.00735	14-Jul-06	602	0.0000
186	21-Jul-06	73.52	-0.00635	21-Jul-06	577	-0.0415
187	28-Jul-06	73.53	0.00014	28-Jul-06	576.5	-0.0009
188	4-Aug-06	76.32	0.03794	4-Aug-06	577.5	0.0017
189	11-Aug-06	76.98	0.00865	11-Aug-06	551	-0.0459
190	18-Aug-06	72.35	-0.06015	18-Aug-06	547	-0.0073
191	25-Aug-06	71.92	-0.00594	25-Aug-06	543	-0.0073
192	1-Sep-06	68.35	-0.04964	1-Sep-06	538.5	-0.0083
193	8-Sep-06	65.76	-0.03789	8-Sep-06	537	-0.0028
194	15-Sep-06	61.38	-0.06661	15-Sep-06	549.75	0.0237
195	22-Sep-06	60.23	-0.01874	22-Sep-06	549.25	-0.0009
196	29-Sep-06	58.76	-0.02441	29-Sep-06	547.5	-0.0032
197	6-Oct-06	57.15	-0.02740	6-Oct-06	564	0.0301
198	13-Oct-06	58.33	0.02065	13-Oct-06	591.5	0.0488
199	20-Oct-06	58.51	0.00309	20-Oct-06	606.5	0.0254
200	27-Oct-06	57.73	-0.01333	27-Oct-06	635.5	0.0478
201	3-Nov-06	56.48	-0.02165	3-Nov-06	649	0.0212
202	10-Nov-06	58.02	0.02727	10-Nov-06	651.25	0.0035
203	17-Nov-06	57.85	-0.00293	17-Nov-06	660.5	0.0142
204	24-Nov-06	59.2	0.02334	24-Nov-06	684.25	0.0360
205	1-Dec-06	62.59	0.05726	1-Dec-06	677	-0.0106
206	8-Dec-06	63.63	0.01662	8-Dec-06	656	-0.0310
207	15-Dec-06	62.56	-0.01682	15-Dec-06	657.5	0.0023
208	22-Dec-06	62.44	-0.00192	22-Dec-06	659.5	0.0030
209	29-Dec-06	59.69	-0.04404	29-Dec-06	683.5	0.0364
210	5-Jan-07	55.63	-0.06802	5-Jan-07	668	-0.0227
211	12-Jan-07	51.79	-0.06903	12-Jan-07	706	0.0569
212	19-Jan-07	51.4	-0.00753	19-Jan-07	717.75	0.0166
213	26-Jan-07	54.87	0.06751	26-Jan-07	710.5	-0.0101
214	2-Feb-07	55.92	0.01914	2-Feb-07	736.75	0.0369
215	9-Feb-07	57.9	0.03541	9-Feb-07	749.25	0.0170
216	16-Feb-07	55.54	-0.04076	16-Feb-07	767	0.0237
217	23-Feb-07	58.16	0.04717	23-Feb-07	778.25	0.0147

218	2-Mar-07	60.62	0.04230	2-Mar-07	739	-0.0504
219	9-Mar-07	60.35	-0.00445	9-Mar-07	746.5	0.0101
220	16-Mar-07	60.87	0.00862	16-Mar-07	753.5	0.0094
221	23-Mar-07	61.09	0.00361	23-Mar-07	769.5	0.0212
222	30-Mar-07	66.1	0.08201	30-Mar-07	761.25	-0.0107
223	6-Apr-07	68.55	0.03707	6-Apr-07	760.5	-0.0010
224	13-Apr-07	68.2	-0.00511	13-Apr-07	738	-0.0296
225	20-Apr-07	66.21	-0.02918	20-Apr-07	723.25	-0.0200
226	27-Apr-07	67.39	0.01782	27-Apr-07	723	-0.0003
227	4-May-07	66.04	-0.02003	4-May-07	733.5	0.0145
228	11-May-07	63.91	-0.03225	11-May-07	750	0.0225
229	18-May-07	67.55	0.05696	18-May-07	792.5	0.0567
230	25-May-07	70.65	0.04589	25-May-07	812.5	0.0252
231	1-Jun-07	68.45	-0.03114	1-Jun-07	817.5	0.0062
232	8-Jun-07	71.23	0.04061	8-Jun-07	821.5	0.0049
233	15-Jun-07	69.89	-0.01881	15-Jun-07	847.25	0.0313
234	22-Jun-07	71.78	0.02704	22-Jun-07	797	-0.0593
235	29-Jun-07	71.76	-0.00028	29-Jun-07	850	0.0665
236	6-Jul-07	74.79	0.04222	6-Jul-07	864.75	0.0174
237	13-Jul-07	77.76	0.03971	13-Jul-07	908.75	0.0509
238	20-Jul-07	78.24	0.00617	20-Jul-07	850.25	-0.0644
239	27-Jul-07	76.66	-0.02019	27-Jul-07	815.5	-0.0409
240	3-Aug-07	76.34	-0.00417	3-Aug-07	839	0.0288
241	10-Aug-07	70.7	-0.07388	10-Aug-07	850	0.0131
242	17-Aug-07	70.09	-0.00863	17-Aug-07	811.5	-0.0453
243	24-Aug-07	68.44	-0.02354	24-Aug-07	849	0.0462
244	31-Aug-07	70.55	0.03083	31-Aug-07	868	0.0224
245	7-Sep-07	75.06	0.06393	7-Sep-07	891	0.0265
246	14-Sep-07	76.89	0.02438	14-Sep-07	941	0.0561
247	21-Sep-07	78.17	0.01665	21-Sep-07	979	0.0404
248	28-Sep-07	78.15	-0.00026	28-Sep-07	991.25	0.0125
249	5-Oct-07	77.98	-0.00218	5-Oct-07	940.5	-0.0512
250	12-Oct-07	78.85	0.01116	12-Oct-07	976.75	0.0385
251	19-Oct-07	84.29	0.06899	19-Oct-07	983.25	0.0067
252	26-Oct-07	83.72	-0.00676	26-Oct-07	995.5	0.0125
253	2-Nov-07	90.42	0.08003	2-Nov-07	1001.75	0.0063
254	9-Nov-07	93.54	0.03451	9-Nov-07	1043	0.0412
255	16-Nov-07	90.74	-0.02993	16-Nov-07	1077.75	0.0333
256	23-Nov-07	94.01	0.03604	23-Nov-07	1100.25	0.0209
257	30-Nov-07	92.16	-0.01968	30-Nov-07	1080	-0.0184
258	7-Dec-07	89.09	-0.03331	7-Dec-07	1119.75	0.0368
259	14-Dec-07	90.24	0.01291	14-Dec-07	1157	0.0333
260	21-Dec-07	90.82	0.00643	21-Dec-07	1177.5	0.0177
261	28-Dec-07	94.39	0.03931	28-Dec-07	1207.75	0.0257
262	4-Jan-08	96.5	0.02235	4-Jan-08	1249	0.0342
263	11-Jan-08	94.4	-0.02176	11-Jan-08	1286	0.0296
264	18-Jan-08	90.03	-0.04629	18-Jan-08	1264	-0.0171
265	25-Jan-08	88.46	-0.01744	25-Jan-08	1243	-0.0166
266	1-Feb-08	91.77	0.03742	1-Feb-08	1287.25	0.0356
267	8-Feb-08	89.88	-0.02059	8-Feb-08	1339	0.0402
268	15-Feb-08	94.98	0.05674	15-Feb-08	1373.75	0.0260
269	22-Feb-08	97.13	0.02264	22-Feb-08	1420	0.0337
270	29-Feb-08	99.1	0.02028	29-Feb-08	1522	0.0718
271	7-Mar-08	101.9	0.02825	7-Mar-08	1341	-0.1189
272	14-Mar-08	107.69	0.05682	14-Mar-08	1340	-0.0007
273	21-Mar-08	103.05	-0.04309	21-Mar-08	1207	-0.0993
274	28-Mar-08	102.05	-0.00970	28-Mar-08	1267.25	0.0499

275	4-Apr-08	100.88	-0.01146	4-Apr-08	1277	0.0077
276	11-Apr-08	106.6	0.05670	11-Apr-08	1332.5	0.0435
277	18-Apr-08	110.42	0.03583	18-Apr-08	1361.5	0.0218
278	25-Apr-08	114.34	0.03550	25-Apr-08	1325.75	-0.0263
279	2-May-08	111.98	-0.02064	2-May-08	1292.5	-0.0251
280	9-May-08	119.84	0.07019	9-May-08	1349.5	0.0441
281	16-May-08	122.58	0.02286	16-May-08	1378	0.0211
282	23-May-08	126.47	0.03173	23-May-08	1368	-0.0073
283	30-May-08	128.76	0.01811	30-May-08	1363.5	-0.0033
284	6-Jun-08	126.33	-0.01887	6-Jun-08	1457.5	0.0689
285	13-Jun-08	134.12	0.06166	13-Jun-08	1560	0.0703
286	20-Jun-08	132.08	-0.01521	20-Jun-08	1532.5	-0.0176
287	27-Jun-08	135.54	0.02620	27-Jun-08	1581.5	0.0320
288	4-Jul-08	141.07	0.04080	4-Jul-08	1658	0.0484
289	11-Jul-08	137.43	-0.02580	11-Jul-08	1630.5	-0.0166
290	18-Jul-08	135.05	-0.01732	18-Jul-08	1470	-0.0984
291	25-Jul-08	126.7	-0.06183	25-Jul-08	1398.75	-0.0485
292	1-Aug-08	124.43	-0.01792	1-Aug-08	1357.75	-0.0293
293	8-Aug-08	116.56	-0.06325	8-Aug-08	1199	-0.1169
294	15-Aug-08	110.16	-0.05491	15-Aug-08	1211.5	0.0104
295	22-Aug-08	111.66	0.01362	22-Aug-08	1321	0.0904
296	29-Aug-08	112.4	0.00663	29-Aug-08	1332	0.0083
297	5-Sep-08	103.69	-0.07749	5-Sep-08	1180	-0.1141
298	12-Sep-08	97.28	-0.06182	12-Sep-08	1490	0.2627
299	19-Sep-08	89.35	-0.08152	19-Sep-08	1143.5	-0.2326
300	26-Sep-08	100.91	0.12938	26-Sep-08	1164	0.0179
301	3-Oct-08	91.9	-0.08929	3-Oct-08	992	-0.1478
302	10-Oct-08	80.98	-0.11882	10-Oct-08	910	-0.0827
303	17-Oct-08	69.28	-0.14448	17-Oct-08	894	-0.0176
304	24-Oct-08	64.4	-0.07044	24-Oct-08	863.75	-0.0338
305	31-Oct-08	60.61	-0.05885	31-Oct-08	925.25	0.0712
306	7-Nov-08	59.43	-0.01947	7-Nov-08	911.75	-0.0146
307	14-Nov-08	53.27	-0.10365	14-Nov-08	878	-0.0370
308	21-Nov-08	47.79	-0.10287	21-Nov-08	840	-0.0433
309	28-Nov-08	48.53	0.01548	28-Nov-08	883	0.0512
310	5-Dec-08	43.7	-0.09953	5-Dec-08	783.5	-0.1127
311	12-Dec-08	41.01	-0.06156	12-Dec-08	854	0.0900
312	19-Dec-08	41.71	0.01707	19-Dec-08	868.25	0.0167
313	26-Dec-08	35.38	-0.15176	26-Dec-08	951.75	0.0962
314	2-Jan-09	37.04	0.04692	2-Jan-09	970	0.0192
315	9-Jan-09	45.25	0.22165	9-Jan-09	1037.5	0.0696
316	16-Jan-09	42.38	-0.06343	16-Jan-09	1020	-0.0169
317	23-Jan-09	41.67	-0.01675	23-Jan-09	1009	-0.0108
318	30-Jan-09	44.2	0.06072	30-Jan-09	980	-0.0287
319	6-Feb-09	43.64	-0.01267	6-Feb-09	1001	0.0214
320	13-Feb-09	45.59	0.04468	13-Feb-09	955.5	-0.0455
321	20-Feb-09	40.91	-0.10265	20-Feb-09	862.5	-0.0973
322	27-Feb-09	42.68	0.04327	27-Feb-09	874.5	0.0139
323	6-Mar-09	43.86	0.02765	6-Mar-09	879	0.0051
324	13-Mar-09	43.98	0.00274	13-Mar-09	882.5	0.0040
325	20-Mar-09	46.43	0.05571	20-Mar-09	952	0.0788
326	27-Mar-09	51.46	0.10834	27-Mar-09	917	-0.0368
327	3-Apr-09	48.49	-0.05771	3-Apr-09	995.5	0.0856
328	10-Apr-09	51.48	0.06166	10-Apr-09	1007	0.0116
329	17-Apr-09	51.59	0.00214	17-Apr-09	1051	0.0437
330	24-Apr-09	48.97	-0.05079	24-Apr-09	1040.25	-0.0102
331	1-May-09	49.92	0.01940	1-May-09	1102	0.0594

332	8-May-09	54.83	0.09836	8-May-09	1134	0.0290
333	15-May-09	56.39	0.02845	15-May-09	1130.5	-0.0031
334	22-May-09	57.89	0.02660	22-May-09	1166	0.0314
335	29-May-09	62.2	0.07445	29-May-09	1184	0.0154
336	5-Jun-09	67.17	0.07990	5-Jun-09	1225.5	0.0351
337	12-Jun-09	69.88	0.04035	12-Jun-09	1245.5	0.0163
338	19-Jun-09	69.68	-0.00286	19-Jun-09	1179	-0.0534
339	26-Jun-09	67.58	-0.03014	26-Jun-09	1201	0.0187
340	3-Jul-09	68.03	0.00666	3-Jul-09	1243	0.0350
341	10-Jul-09	60.39	-0.11230	10-Jul-09	1128.25	-0.0923
342	17-Jul-09	61.11	0.01192	17-Jul-09	1009.5	-0.1053
343	24-Jul-09	66.56	0.08918	24-Jul-09	1021	0.0114
344	31-Jul-09	68.6	0.03065	31-Jul-09	1134	0.1107
345	7-Aug-09	73.99	0.07857	7-Aug-09	1184.5	0.0445
346	14-Aug-09	72.9	-0.01473	14-Aug-09	1100	-0.0713
347	21-Aug-09	71.52	-0.01893	21-Aug-09	1023	-0.0700
348	28-Aug-09	72.33	0.01133	28-Aug-09	1135.75	0.1102
349	4-Sep-09	67.6	-0.06539	4-Sep-09	961	-0.1539
350	11-Sep-09	69.17	0.02322	11-Sep-09	984.5	0.0245
351	18-Sep-09	68.85	-0.00463	18-Sep-09	941	-0.0442
352	25-Sep-09	66.96	-0.02745	25-Sep-09	926	-0.0159
353	2-Oct-09	65.9	-0.01583	2-Oct-09	885	-0.0443
354	9-Oct-09	67.87	0.02989	9-Oct-09	964	0.0893
355	16-Oct-09	72.29	0.06512	16-Oct-09	977.5	0.0140
356	23-Oct-09	77.24	0.06847	23-Oct-09	1005.5	0.0286
357	30-Oct-09	76.07	-0.01515	30-Oct-09	978	-0.0273
358	6-Nov-09	76.6	0.00697	6-Nov-09	948	-0.0307
359	13-Nov-09	76.25	-0.00457	13-Nov-09	984	0.0380
360	20-Nov-09	77.04	0.01036	20-Nov-09	1046	0.0630
361	27-Nov-09	76.52	-0.00675	27-Nov-09	1053	0.0067
362	4-Dec-09	77.78	0.01647	4-Dec-09	1043	-0.0095
363	11-Dec-09	73.14	-0.05966	11-Dec-09	1035	-0.0077
364	18-Dec-09	71.8	-0.01832	18-Dec-09	1012	-0.0222
365	25-Dec-09	73.35	0.02159	25-Dec-09	999.5	-0.0124
366	1-Jan-10	77.19	0.05235	1-Jan-10	1039.75	0.0403
367	8-Jan-10	79.82	0.03407	8-Jan-10	1013	-0.0257
368	15-Jan-10	78.31	-0.01892	15-Jan-10	974	-0.0385
369	22-Jan-10	74.28	-0.05146	22-Jan-10	951.5	-0.0231
370	29-Jan-10	71.88	-0.03231	29-Jan-10	914	-0.0394
371	5-Feb-10	72.54	0.00918	5-Feb-10	913.5	-0.0005
372	12-Feb-10	70.85	-0.02330	12-Feb-10	945	0.0345
373	19-Feb-10	75.8	0.06987	19-Feb-10	945	0.0000
374	26-Feb-10	76.23	0.00567	26-Feb-10	951	0.0063
375	5-Mar-10	77.86	0.02138	5-Mar-10	934.75	-0.0171
376	12-Mar-10	79.36	0.01927	12-Mar-10	926	-0.0094
377	19-Mar-10	79.05	-0.00391	19-Mar-10	961.75	0.0386
378	26-Mar-10	78.38	-0.00848	26-Mar-10	952	-0.0101
379	2-Apr-10	80.59	0.02820	2-Apr-10	942	-0.0105
380	9-Apr-10	83.88	0.04082	9-Apr-10	952.25	0.0109
381	16-Apr-10	85.23	0.01609	16-Apr-10	985.25	0.0347
382	23-Apr-10	84.61	-0.00727	23-Apr-10	1000	0.0150
383	30-Apr-10	85.98	0.01619	30-Apr-10	989.5	-0.0105
384	7-May-10	82.5	-0.04047	7-May-10	951.25	-0.0387
385	14-May-10	78.32	-0.05067	14-May-10	948	-0.0034
386	21-May-10	72.17	-0.07852	21-May-10	941	-0.0074
387	28-May-10	70.79	-0.01912	28-May-10	937.75	-0.0035
388	4-Jun-10	72.71	0.02712	4-Jun-10	935	-0.0029

389	11-Jun-10	72.76	0.00069	11-Jun-10	946.25	0.0120
390	18-Jun-10	76.22	0.04755	18-Jun-10	961	0.0156
391	25-Jun-10	76.64	0.00551	25-Jun-10	957	-0.0042
392	2-Jul-10	73.86	-0.03627	2-Jul-10	962.75	0.0060
393	9-Jul-10	73.95	0.00122	9-Jul-10	1025.5	0.0652
394	16-Jul-10	75.7	0.02366	16-Jul-10	1019.5	-0.0059
395	23-Jul-10	76.64	0.01242	23-Jul-10	1017	-0.0025
396	30-Jul-10	77.24	0.00783	30-Jul-10	1052.5	0.0349
397	6-Aug-10	82.69	0.07056	6-Aug-10	1059	0.0062
398	13-Aug-10	78.21	-0.05418	13-Aug-10	1052	-0.0066
399	20-Aug-10	74.94	-0.04181	20-Aug-10	1009.25	-0.0406
400	27-Aug-10	72.82	-0.02829	27-Aug-10	1022	0.0126
401	3-Sep-10	75.41	0.03557	3-Sep-10	1029.75	0.0076
402	10-Sep-10	77.17	0.02334	10-Sep-10	1023.5	-0.0061
403	17-Sep-10	78.44	0.01646	17-Sep-10	1069	0.0445
404	24-Sep-10	78.38	-0.00076	24-Sep-10	1126	0.0533
405	1-Oct-10	79.82	0.01837	1-Oct-10	1057	-0.0613
406	8-Oct-10	83.87	0.05074	8-Oct-10	1135	0.0738
407	15-Oct-10	83.11	-0.00906	15-Oct-10	1185	0.0441
408	22-Oct-10	81.43	-0.02021	22-Oct-10	1199.5	0.0122
409	29-Oct-10	82.25	0.01007	29-Oct-10	1226	0.0221
410	5-Nov-10	85.6	0.04073	5-Nov-10	1273.5	0.0387
411	12-Nov-10	87.43	0.02138	12-Nov-10	1263	-0.0082
412	19-Nov-10	83.94	-0.03992	19-Nov-10	1201.5	-0.0487
413	26-Nov-10	83.51	-0.00512	26-Nov-10	1238.5	0.0308
414	3-Dec-10	88.1	0.05496	3-Dec-10	1300.25	0.0499
415	10-Dec-10	90.25	0.02440	10-Dec-10	1273	-0.0210
416	17-Dec-10	90.91	0.00731	17-Dec-10	1298.75	0.0202
417	24-Dec-10	92.9	0.02189	24-Dec-10	1349.5	0.0391
418	31-Dec-10	93.17	0.00291	31-Dec-10	1393.75	0.0328
419	7-Jan-11	94.72	0.01664	7-Jan-11	1357.75	-0.0258
420	14-Jan-11	97.09	0.02502	14-Jan-11	1406.5	0.0359
421	21-Jan-11	97.34	0.00257	21-Jan-11	1412.25	0.0041
422	28-Jan-11	96.62	-0.00740	28-Jan-11	1398	-0.0101
423	4-Feb-11	100.36	0.03871	4-Feb-11	1433.5	0.0254
424	11-Feb-11	99.9	-0.00458	11-Feb-11	1416	-0.0122
425	18-Feb-11	102.81	0.02913	18-Feb-11	1368	-0.0339
426	25-Feb-11	110.49	0.07470	25-Feb-11	1365.5	-0.0018
427	4-Mar-11	114.53	0.03656	4-Mar-11	1407.75	0.0309
428	11-Mar-11	114.45	-0.00070	11-Mar-11	1326.5	-0.0577
429	18-Mar-11	112.67	-0.01555	18-Mar-11	1362.5	0.0271
430	25-Mar-11	115.41	0.02432	25-Mar-11	1358.25	-0.0031
431	1-Apr-11	116.49	0.00936	1-Apr-11	1393.75	0.0261
432	8-Apr-11	123.03	0.05614	8-Apr-11	1392.25	-0.0011
433	15-Apr-11	123.57	0.00439	15-Apr-11	1331.75	-0.0435
434	22-Apr-11	122.74	-0.00672	22-Apr-11	1380.5	0.0366
435	29-Apr-11	125.36	0.02135	29-Apr-11	1392.75	0.0089
436	6-May-11	119.56	-0.04627	6-May-11	1325	-0.0486
437	13-May-11	114.53	-0.04207	13-May-11	1337	0.0091
438	20-May-11	112.02	-0.02192	20-May-11	1380.25	0.0323
439	27-May-11	113.41	0.01241	27-May-11	1379.75	-0.0004
440	3-Jun-11	115.68	0.02002	3-Jun-11	1414.5	0.0252
441	10-Jun-11	117.73	0.01772	10-Jun-11	1387.25	-0.0193
442	17-Jun-11	116.79	-0.00798	17-Jun-11	1333	-0.0391
443	24-Jun-11	110.18	-0.05660	24-Jun-11	1320.25	-0.0096
444	1-Jul-11	109.03	-0.01044	1-Jul-11	1322.25	0.0015
445	8-Jul-11	115.39	0.05833	8-Jul-11	1352	0.0225

```
446   15-Jul-11   117.72      0.02019     15-Jul-11   1385.75     0.0250
447   22-Jul-11   118.2       0.00408     22-Jul-11   1380.25     -0.0040
448   29-Jul-11   117.7       -0.00423    29-Jul-11   1354.25     -0.0188
449   5-Aug-11    112.65      -0.04291    5-Aug-11    1331.5      -0.0168
450   12-Aug-11   105.3       -0.06525    12-Aug-11   1337        0.0041
451   19-Aug-11   109.54      0.04027     19-Aug-11   1359.75     0.0170
452   26-Aug-11   111.06      0.01388     26-Aug-11   1414.75     0.0404
453   2-Sep-11    116.11      0.04547     2-Sep-11    1436        0.0150
454   9-Sep-11    115.97      -0.00121    9-Sep-11    1416.5      -0.0136
455   16-Sep-11   114.98      -0.00854    16-Sep-11   1355.5      -0.0431
456   23-Sep-11   111.98      -0.02609    23-Sep-11   1258        -0.0719
457   30-Sep-11   107.69      -0.03831    30-Sep-11   1179        -0.0628
458   7-Oct-11    104.03      -0.03399    7-Oct-11    1158.01     -0.0178
459   14-Oct-11   111.59      0.07267     14-Oct-11   1270        0.0967
460   21-Oct-11   111.47      -0.00108    21-Oct-11   1212.01     -0.0457
461   28-Oct-11   111.33      -0.00126    28-Oct-11   1217        0.0041
462   4-Nov-11    109.84      -0.01338    4-Nov-11    1212.01     -0.0041
463   11-Nov-11   114.68      0.04406     11-Nov-11   1166        -0.0380
464   18-Nov-11   110.69      -0.03479    18-Nov-11   1168.01     0.0017
465   25-Nov-11   106.67      -0.03632    25-Nov-11   1106.01     -0.0531
466   2-Dec-11    110.05      0.03169     2-Dec-11    1135.02     0.0262
467   9-Dec-11    109.31      -0.00672    9-Dec-11    1107        -0.0247
468   16-Dec-11   106.26      -0.02790    16-Dec-11   1130        0.0208
469   23-Dec-11   107.72      0.01374     23-Dec-11   1163        0.0292
470   30-Dec-11   107.51      -0.00195    30-Dec-11   1198.01     0.0301
471   6-Jan-12    112.51      0.04651     6-Jan-12    1189.01     -0.0075
472   13-Jan-12   111.78      -0.00649    13-Jan-12   1160        -0.0244
473   20-Jan-12   109.6       -0.01950    20-Jan-12   1187        0.0233
474   27-Jan-12   109.18      -0.00383    27-Jan-12   1219        0.0270
475   3-Feb-12    111.2       0.01850     3-Feb-12    1232.01     0.0107
476   10-Feb-12   117.21      0.05405     10-Feb-12   1229        -0.0024
477   17-Feb-12   119.79      0.02201     17-Feb-12   1267.01     0.0309
478   24-Feb-12   123.34      0.02964     24-Feb-12   1279        0.0095
479   2-Mar-12    124.88      0.01249     2-Mar-12    1328.01     0.0383
480   9-Mar-12    126.62      0.01393     9-Mar-12    1331.02     0.0023
481   16-Mar-12   126.22      -0.00316    16-Mar-12   1374        0.0323

;
;
run;

ods listing close;
data work.kaili(drop=date2);
      set work.kaili(rename=(date1=date));
run;
proc surveyselect data=work.kaili out=temp2 reps = 481 rate=1 method=srs
noprint;
run;

proc corr data=temp2 kendall;
      by replicate;
      var oilprice sbprice;
      where (replicate ge 2) and (id le replicate);
      ods output kendallcorr = ktb;
run;

proc sql;
```

```
      create table ktb2 as
      select replicate, sbprice as ktb
      from ktb
      where variable = "oilprice"
      ;

      create table ktb3 as
      select ktb as tau_n
      from ktb2
      where replicate=481
      ;

      create table ktb4 as
      select ktb2.replicate, ktb2.ktb, ktb3.tau_n
      from ktb2, ktb3
      ;

quit;

data work.ktb_results;
      set work.ktb4;

      t = replicate*abs(ktb-tau_n)/sqrt(481);
run;

proc sort data=work.ktb_results;
      by descending t;
run;


proc iml;

      use work.kaili;
            read all var {id oilprice sbprice} into xymat;
      close work.kaili;

      n = nrow(xymat);

      call sort(xymat,2);

      Funi = j(n,1,.);
      do i = 1 to n;
            Funi[i] = i/n;
      end;

      xymat = xymat||Funi;

      call sort(xymat,3);

      xymat = xymat||Funi;


      call sort(xymat,1);

      Fxy = j(n,1,.);
```

```
	do i = 1 to n;

		Fxy[i] = sum(xymat[,2] <= xymat[i,2] & xymat[,3] <=
xymat[i,3])/n;

	end;

	xymat = xymat||Fxy;


	create work.cdfs from xymat;
		append from xymat;
	close work.cdfs;



quit;

data work.ktb5;

	set work.ktb3;

	replicate = 1;
	ktb = .;
run;

data work.ktb4;
	retain replicate ktb tau_n;
	set work.ktb5 work.ktb4;
run;

data work.psi;
	merge work.ktb4 work.cdfs(rename=(col1=Replicate col2=oilprice
col3=SBprice col4=Fx col5=Fy col6=Fxy));
	by replicate;

	psi = 2*Fxy - Fx - Fy + 1 -tau_n;
run;

proc iml;

	use work.psi;
		read all var {psi} into x;
	close work.psi;

	n = nrow(x);
	sum3 = j(n-1,1,.);

	do j = 1 to n-1;
		temp = 0;
		do i = 1 to n-j;

		temp = temp + x[i]*x[i+j];
		if i = n-j then sum3[j] = temp;

	end;
end;
```

```
sum2=0;

do j = 1 to n-1;

if abs(j/floor(2*n**(1/3))) <= 1 then kernal = (1-
(j/floor(2*n**(1/3)))**2)**2;
      else kernal = 0;

      sum2 = sum2 + kernal*sum3[j];

      end;
      sum1 = x`*x;

      D2 = sum1/n + 2* sum2/n;

      print D2;

      use work.ktb_results;
            read var{t} into t;
      close work.ktb_results;

      print t;

TS = t/2/sqrt(D2);

      print TS;

            F=0;

      do i = 1 to 100000;

            F = F + (-1)**i * exp(-2*(i**2)*(TS**2));

      end;

      p = -F;

      print p;

quit;

data work.ktb_output;
      set work.ktb_results(drop=ktb tau_n);
      if _N_ gt 1 then delete;
run;
```

APPENDIX B

SAS Code for Simulations

```
libname Kaili "C:\Users\ld01373\Desktop\Kaili";

%let n1 = 1000;
%let n2 = 500;
%let r1 = 0.0;
%let r2 = 0.7;
%let nsim = 3;

%macro loop;
      %do i = 1 %to &nsim;
      dm 'log;clear;output;clear;';

data random (keep=id x y);

      c1 = sqrt(1-&r1**2);
      c2 = sqrt(1-&r2**2);
      do id = 1 to &n1 + &n2;
      x=rannor(0);
            if id le &n1 then y=&r1*x+c1*rannor(0);
            else y = &r2*x+c2*rannor(0);
      output;
      end;
      run;


proc surveyselect data=work.random out=temp2 reps = %eval(&n1 + &n2) rate=1
method=srs noprint;
run;

proc corr data=temp2 kendall;
      by replicate;
      var x y;
      where (replicate ge 2) and (id le replicate);
      ods output kendallcorr = ktb;
run;

proc sql;
      create table ktb2 as
      select replicate, (y+1)/2 as ktb
      from ktb
      where variable = "x"
      ;

      create table ktb3 as
      select ktb as tau_n
      from ktb2
      where replicate=&n1 + &n2
      ;
```

```
	create table ktb4 as
	select ktb2.replicate, ktb2.ktb, ktb3.tau_n
	from ktb2, ktb3
	;

quit;

data work.ktb_results;
	set work.ktb4;

	t = replicate*abs(ktb-tau_n)/sqrt(&n1 + &n2);
run;

proc sort data=work.ktb_results;
	by descending t;
run;


proc iml;

	use work.random;
		read all var {id x y} into xymat;
	close work.random;

	n = nrow(xymat);

	call sort(xymat,2);

	Funi = j(n,1,.);
	do i = 1 to n;
		Funi[i] = i/n;
	end;

	xymat = xymat||Funi;

	call sort(xymat,3);

	xymat = xymat||Funi;


	call sort(xymat,1);

	Fxy = j(n,1,.);


	do i = 1 to n;

		Fxy[i] = sum(xymat[,2] <= xymat[i,2] & xymat[,3] <=
xymat[i,3])/n;

	end;

	xymat = xymat||Fxy;


	create work.cdfs from xymat;
		append from xymat;
```

```
      close work.cdfs;



quit;

data work.ktb5;

      set work.ktb3;

      replicate = 1;
      ktb = .;
run;

data work.ktb4;
      retain replicate ktb tau_n;
      set work.ktb5 work.ktb4;
run;

data work.psi;
      merge work.ktb4 work.cdfs(rename=(col1=Replicate col2=x col3=y col4=Fx
col5=Fy col6=Fxy));
      by replicate;

      psi = 2*Fxy - Fx - Fy + 1 -tau_n;
run;

proc iml;

      use work.psi;
            read all var {psi} into x;
      close work.psi;

      n = nrow(x);
      sum3 = j(n-1,1,.);

      do j = 1 to n-1;
            temp = 0;
            do i = 1 to n-j;

            temp = temp + x[i]*x[i+j];
            if i = n-j then sum3[j] = temp;

      end;
end;

sum2=0;

do j = 1 to n-1;

if abs(j/floor(2*n**(1/3))) <= 1 then kernal = (1-
(j/floor(2*n**(1/3)))**2)**2;
      else kernal = 0;

      sum2 = sum2 + kernal*sum3[j];

      end;
```

```
      sum1 = x`*x;

      D2 = sum1/n + 2* sum2/n;



      use work.ktb_results;
            read var{t} into t;
      close work.ktb_results;



TS = t/2/sqrt(D2);



            F=0;

      do i = 1 to 100000;

            F = F + (-1)**i * exp(-2*(i**2)*(TS**2));

      end;

      p = -F;

results_mat = D2||TS||p;
results_names="D2"||"TS"||"p-value";

create work.iml_results from results_mat [colname=results_names];
      append from results_mat;
close work.iml_results;

      quit;




data
data work.final;
      set work.ktb_results;
      set work.iml_results;
      n1 = &n1;
      n2 = &n2;
      r1 = &r1;
      r2 = &r2;

run;

proc append base=Kaili.Out data=work.final;
run;

%end;
%mend;

%loop;
```